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Excitation of Waves in a Wedge-Shaped Region

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Abstract—Solution of the problem on an impedance wedge is studied by the Wiener–Hopf method. The range of applicability of the solution is extended to the wedge angles exceeding π. A procedure is proposed for solving the problem in the case of the wedge angle being equal to π/m.

INTRODUCTION

The problem on the diffraction of a plane acoustic wave by a wedge with impedance boundary conditions was solved by G.D. Malyuzhinets [1, 2]. The solution was obtained by using functional equations in the Sommerfeld transform. Later, a detailed theory of such equations was developed [3–5], which allowed one to solve inhomogeneous equations, i.e., to take into account the external forces.

It should be noted that the Malyuzhinets theory of functional equations is fairly complicated. It is based on special kinds of functions introduced by Malyuzhinets and determined by their integral representation or in the form of an infinite product. Because of the technical difficulties arising in the practical application of this theory, the possibilities of putting it into practice are limited. However, today, the ideas put forward by Malyuzhinets have received further development in a number of publications [6–8].

This paper presents the development of another approach to the problems of diffraction in wedge-shaped regions [9, 10]. The approach is based on the derivation of functional equations related to those proposed by Malyuzhinets. However, these equations can be solved by applying the Wiener–Hopf method. It should be noted that similar equations were derived earlier by Gautesen [11].

The Wiener–Hopf method is widely used in radiowave physics, and its theory is sufficiently complete [12]. On the other hand, functional equations similar to those studied below can be easily derived for any two-dimensional region with a piecewise linear boundary. These two facts should facilitate the solution of two-dimensional problems concerned with the diffraction and propagation of waves. Specifically, the above-mentioned approach made it possible to describe the field excitation in a two-dimensional region represented by an equilateral triangle with impedance boundary conditions [13].

This paper develops the results obtained in paper [9] in two respects. First, the derivation of the functional equation and the application of the factorization method are considered for a wedge of angle θ greater than π. (The results obtained in [9] refer only to $\theta < \pi$.) In this case, the same functional equation is valid, and the region of the *a priori* analyticity of the sought-after function is extended (from the half-band $0 < \mathrm{Re}\,\varphi < \pi$, $\mathrm{Im}\varphi > 0$ to the half-band $0 < \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi > 0$), although it is possible that, in the extended region, the sought-after function has a pole corresponding to the Brewster angle of the boundaries. The extension of the region of analyticity of the unknown functions is necessary for a successful application of the Wiener–Hopf method.

Second, the Wiener–Hopf method is used for calculating the wave field formed in a wedge of angle π/m. The conventional procedure of applying the Wiener–Hopf method to a functional equation includes two basic steps. At the first step, a multiplicative factorization of the coefficients involved in the equation is performed. In our case, this step leads to the appearance of Malyuzhinets functions degenerating into combinations of elementary functions at rational wedge angles. At the second step, the right-hand member of the transformed equation is expanded into the sum of functions that are analytical in the complementary regions. Below, it is shown that, at the wedge angles equal to π/m, the initial functional equation can be transformed by the reflection method in such a way that it will require no multiplicative factorization (and, hence, no Malyuzhinets functions will appear). In this case, the unknown function is expressed in the form of a contour integral different from that obtained before [9].

It should be noted that the relation of the Malyuzhinets function to the reflection coefficient characterizing the plane wave reflection from an impedance plane was mentioned in Malyuzhinets' dissertation as well as in some more recent publications (e.g., [14]).

STATEMENT OF THE PROBLEM, DERIVATION OF FUNCTIONAL EQUATIONS, AND SOLUTION IN THE CASE $\theta < \pi$

In this section, we describe the statement of the problem and briefly review the main results obtained in paper [9].

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We consider an angular region (a wedge) filled with liquid or gas. Let the wedge angle be $\theta < \pi$. We assume that the Helmholtz equation holds in this region for some function $u(x, y)$:

$$\Delta u + k_0^2 u = 0. \quad (1)$$

The consideration will be performed (where necessary) in polar coordinates (r, α). We select the time dependence in the form $e^{i\omega t}$. A wave propagating along the x axis in the positive direction has the form e^{-ik_0x}. We assume that nonlinear impedance boundary conditions are fulfilled at the wedge boundaries:

$$\frac{\partial u}{\partial n} - iuk_0\sin\beta = \Phi_{0,1}(r), \quad (2)$$

where n is the inner normal to the boundary and $\sin\beta$ is the constant related to the Brewster angle of the surface ($\mathrm{Re}\beta \in (0, \pi/2]$). The functions $\Phi_{0,1}(r)$ represent the external forces applied to the faces of the wedge (the subscript indicates the face number).

We seek the wave field formed by the sources located at the wedge boundary near the tip of the wedge. We impose the condition at infinity and the Meixner conditions at the tip (these conditions imply the absence of energy sources near the tip of the wedge).

Let us repeat the main statements of paper [9] that will allow us to solve the above-stated problem.

Applying the Green formula to the angular region II and selecting an auxiliary solution in the form of a plane wave propagating into the bulk of the wedge, we obtain the functional equation

$$(\sin\beta + \sin\varphi)\hat{u}_0(\varphi) + (\sin\beta + \sin(\theta - \varphi))\hat{u}_1(\theta - \varphi) = \hat{\Phi}_0(\varphi) + \hat{\Phi}_1(\theta - \varphi), \quad (3)$$

where

$$\hat{u}_0(\varphi) = \int_0^\infty u(r, 0)e^{-ik_0 r\cos\varphi}dr,$$

$$\hat{u}_1(\varphi) = \int_0^\infty u(r, \theta)e^{-ik_0 r\cos\varphi}dr, \quad (4)$$

$$\hat{\Phi}_{0,1}(\varphi) = \frac{i}{k_0}\int_0^\infty \Phi_{0,1}(r)e^{-ik_0 r\cos\varphi}dr.$$

Equation (3) is determined at $0 < \varphi < \theta$, but it can be shown that this equation holds at real φ within the interval $\theta - \pi < \varphi < \pi$ and can be analytically extended beyond this interval.

According to formulas (4), the functions $\hat{u}_{1,2}(\varphi)$ are regular in the half-band $0 < \mathrm{Re}\varphi < \pi$, $\mathrm{Im}\varphi > 0$ (the domain of the *a priori* analyticity). Besides, the following identity is evident:

$$\hat{u}_{0,1}(\varphi) = \hat{u}_{0,1}(-\varphi). \quad (5)$$

In addition, by virtue of the conditions fulfilled at the tip, the function $\hat{u}_{0,1}(\varphi)$ decreases at infinity as $\cos^{-1}\varphi$ in the half-band $0 < \mathrm{Re}\varphi < \pi$, $\mathrm{Im}\varphi > 0$. The above-listed conditions are important for the solution of the problem under study. The symmetry condition is in fact an additional functional equation expressing the conditions of the Sommerfeld radiation. The analyticity of the functions within the half-band allows us to eliminate the unnecessary poles (each pole is related to an arriving plane wave). The condition of the decrease at infinity ensures the uniqueness of the solution and the fulfillment of the Meixner conditions at the tip.

The function

$$A(\varphi) = \sin\beta + \sin\varphi,$$

representing the coefficient involved in equation (3) can be factorized at any given $0 < \theta < \pi$ in the following way:

$$A(\varphi) = A^-(\varphi)/A^+(\varphi), \quad (6)$$

where

$$A^+(\varphi) = \frac{1}{\Psi_{\theta/2}(\varphi + \beta - \pi/2 - \theta)\Psi_{\theta/2}(\varphi - \beta + \pi/2 - \theta)},$$

$$A^-(\varphi) = (\sin\beta + \sin\varphi)A^+(\varphi).$$

Here and below, $\Psi_{\theta/2}(z)$ is the Malyuzhinets function.

For the functions $A^-(\varphi)$ and $A^+(\varphi)$, the following identities are true:

$$A^-(\varphi) = A^-(-\varphi), \quad A^+(\varphi) = A^+(2\theta - \varphi). \quad (7)$$

In addition, the functions $A^-(\varphi)$ and $A^+(\varphi)$ have no zeros and no poles in the regions $0 < \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi > 0$ and $0 < \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi < 0$, respectively. Then, equation (3) takes the form

$$A^+(\theta - \varphi)A^-(\varphi)\hat{u}_0(\varphi) + A^-(\theta - \varphi)A^+(\varphi)\hat{u}_1(-\varphi) = A^+(\theta - \varphi)A^+(\varphi)(\hat{\Phi}_0(\varphi) + \hat{\Phi}_1(\theta - \varphi)), \quad (8)$$

where the two terms on the left-hand side have no singularities in the half-bands $0 < \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi > 0$ and $0 < \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi < 0$, respectively. These regions are subdomains of the domains of the *a priori* analyticity of the unknown functions. The first term is a function symmetric about the center $\varphi = 0$, and the second term is a function symmetric about $\varphi = \theta$. Both terms on the left-hand side decrease at infinity.

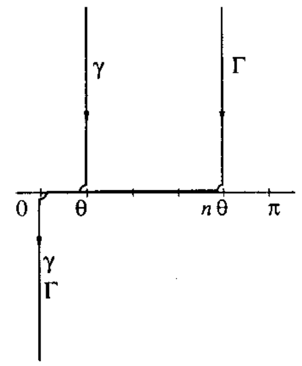

Fig. 1. Integration paths γ and Γ for θ = π/5.

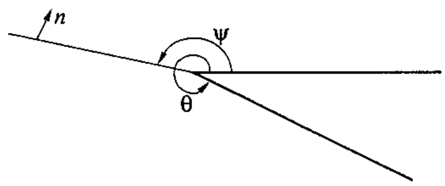

Fig. 2. Obtuse wedge.

The above-mentioned properties of the functions allow us to solve equation (8). In the region $0 < \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi > 0$, the solution can be presented in the form

$$\hat{u}_0(\varphi) = \frac{1}{A^-(\varphi)A^+(\theta-\varphi)} \tag{9}$$
$$\times\int_\gamma \frac{A^+(\varphi')A^+(\theta-\varphi')[\hat{\Phi}_0(\varphi')+\hat{\Phi}_1(\theta-\varphi')]}{2\theta k_0[\cos(\pi\varphi'/\theta)-\cos(\pi\varphi/\theta)]}\sin\left(\frac{\pi\varphi'}{\theta}\right)d\varphi'.$$

The path of integration γ is shown in Fig. 1.

Let us consider formula (9). Suppose that we need to represent a known function $F(\varphi)$ in the form of a sum

$$F(\varphi) = f^+(\varphi) + f^-(\varphi), \tag{10}$$

so that $f^+(\varphi)$ has no singularities in the half-band $0 < \mathrm{Re}\varphi < \Theta$, $\mathrm{Im}\varphi < 0$, and $f^-(\varphi)$ has no singularities in the half-band $0 < \mathrm{Re}\varphi < \Theta$, $\mathrm{Im}\varphi > 0$, and the following equalities are satisfied:

$$f^-(\varphi) = f^-(-\varphi), \quad f^+(\varphi) = f^+(2\Theta-\varphi). \tag{11}$$

The functions $f^-(\varphi)$ and $f^+(\varphi)$ are determined as

$$f^-(\varphi) = \frac{1}{2\Theta i}\int_\Gamma \frac{F(\varphi')\sin(\pi\varphi'/\Theta)d\varphi'}{\cos(\pi\varphi'/\Theta)-\cos(\pi\varphi/\Theta)} \tag{12}$$

at $0 < \mathrm{Re}\varphi < \Theta$, $\mathrm{Im}\varphi > 0$, and

$$f^+(\varphi) = -\frac{1}{2\Theta i}\int_\Gamma \frac{F(\varphi')\sin(\pi\varphi'/\Theta)d\varphi'}{\cos(\pi\varphi'/\Theta)-\cos(\pi\varphi/\Theta)}$$

at $0 < \mathrm{Re}\varphi < \Theta$, $\mathrm{Im}\varphi < 0$. Conditions (10) and (11) allow the determination of $f^+(\varphi)$ and $f^-(\varphi)$ at every φ. The integration path Γ consists of the paths $(\theta + i\infty, \Theta)$, $(\Theta, 0)$, and $(0, -i\infty)$.

Formulas (12) can be derived in the following way. Let us introduce the variable $\alpha = \cos(\pi\varphi/\Theta)$. In terms of this variable, the integration path Γ passes into the real axis α, and the classical problem of expanding a given function into a sum of functions analytical in the upper and lower half-planes is to be solved.

Evidently, the unknown functions are determined correct to an arbitrary integral function, but, because the conditions of the decrease at infinity are imposed on the functions f^+ and f^-, the ambiguity is eliminated.

CASE θ > π

The case of θ > π should be considered separately. Below, it will be demonstrated that, in this case, the same functional equations are satisfied, but the limitations imposed on the analyticity of the unknown functions are somewhat different. This difference is related to the fact that the application of the Wiener–Hopf method requires the functions $\hat{u}_{0,1}(\varphi)$ to be analytical in the region $0 < \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi > 0$, which, in the case under study, is no subdomain of the domain of the *a priori* analyticity.

We divide the angle θ by an imaginary straight line into two parts, so that each part is less than π (Fig. 2). We assume that this line makes an angle ψ with one arm of the angle. We also preset the values of the variable $u(l)$ and its normal derivative at this line (the normal along which we perform the differentiation is shown in Fig. 2).

By applying the second Green formula to each of the two angular regions, we obtain two equations:

$$(\sin\beta + \sin\varphi)\hat{u}_0(\varphi) + \hat{s}(\psi-\varphi) + \sin(\psi-\varphi)\hat{v}(\psi-\varphi) = \hat{\Phi}_0(\varphi), \tag{13}$$

$$(\sin\beta + \sin(\theta-\varphi))\hat{u}_1(\theta-\varphi) - \hat{s}(\psi-\varphi) - \sin(\psi-\varphi)\hat{v}(\psi-\varphi) = \hat{\Phi}_1(\theta-\varphi), \tag{14}$$

where

$$\hat{v}(\varphi) = \int_0^\infty u(r,\psi)e^{-ik_0r\cos\varphi}dr,$$

$$\hat{s}(\varphi) = \frac{1}{ik_0}\int_0^\infty \frac{\partial u(r,\psi)}{\partial n}e^{-ik_0r\cos\varphi}dr.$$

Equations (13) and (14) are determined within the intervals $0 < \varphi < \psi$ and $\psi < \varphi < \theta$, respectively. By an analytical extension, we can make both equations to be determined on a common Riemannian surface. Combining equations (13) and (14), we obtain equation (3).

We note that the functions $\hat{v}(\varphi)$ and $\hat{s}(\varphi)$ are analytical within the half-band $0 < \mathrm{Re}\varphi < \pi$, $\mathrm{Im}\varphi > 0$ and symmetric about the point $\varphi = 0$. Taking this fact into account and using equations (13) and (14), we conclude that the function $\hat{u}_0(\varphi)$ is analytical in the half-band $0 < \mathrm{Re}\varphi < \psi$, $\mathrm{Im}\varphi < 0$, while the function $\hat{u}_1(\varphi)$ is analytical in the half-band $0 < \mathrm{Re}\varphi < \theta - \psi$, $\mathrm{Im}\varphi < 0$. Here, we explicitly allow for the fact that the function $\sin\beta + \sin\varphi$ has no zeros in the aforementioned regions.

Now, we consider equation (3). From this equation, it follows that the functions $\hat{u}_{0,1}(\varphi)$ are analytical in the half-band $\pi \le \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi > 0$ except, maybe, for the simple poles at the points where $\sin\beta + \sin\varphi = 0$. Thus, in the half-band $0 \le \mathrm{Re}\varphi < \theta$, $\mathrm{Im}\varphi > 0$, the functions $\hat{u}_{0,1}(\varphi)$ can have only simple poles corresponding to the zero values of the function $A(\varphi)$. A more detailed analysis shows that, for β having an imaginary component, the functions $\hat{u}_{0,1}(\varphi)$ can be considered as analytical over the integration path γ.

We note that the function $A^-(\varphi)$ has a simple zero at the point where $A(\varphi) = 0$. Hence, the terms standing on the left-hand side of equation (8) satisfy all conditions specified in the previous section. As a result, the solution to this equation will be similar to solution (9).

APPLICATION OF THE REFLECTION METHOD TO THE FUNCTIONAL EQUATION IN THE CASE OF THE WEDGE ANGLE BEING A FRACTION OF π

The procedure of applying the reflection method to functional equations of type (3) consists in the sequential elimination of the unknown functions with the arguments corresponding to the rays reflected from the arms of the angle. In the case of wedge angles equal to π/m, such procedure leads to qualitative changes in the properties of the initial equation.

We write functional equation (3) several times (namely, n times) with the sequential replacement of the argument of the unknown function by the opposite argument:

$$A(\varphi)\hat{u}_0(\varphi) + A(\theta-\varphi)\hat{u}_1(\theta-\varphi) = \hat{\Phi}_0(\varphi) + \hat{\Phi}_1(\theta-\varphi),$$

$$A(2\theta-\varphi)\hat{u}_0(2\theta-\varphi) + A(\varphi-\theta)\hat{u}_1(\varphi-\theta) = \hat{\Phi}_0(2\theta-\varphi) + \hat{\Phi}_1(\varphi-\theta),$$

$$A(\varphi-2\theta)\hat{u}_0(\varphi-2\theta) + A(3\theta-\varphi)\hat{u}_1(3\theta-\varphi) = \hat{\Phi}_0(-2\theta+\varphi) + \hat{\Phi}_1(3\theta-\varphi),$$

$$A(4\theta-\varphi)\hat{u}_0(4\theta-\varphi) + A(\varphi-3\theta)\hat{u}_1(\varphi-3\theta) = \hat{\Phi}_0(4\theta-\varphi) + \hat{\Phi}_1(\varphi-3\theta),$$

$$\ldots$$

Using the symmetry property (5), we exclude the variable $\hat{u}_1(\varphi-\theta)$ from the first two equations and obtain an equation relating $\hat{u}_0(\varphi)$ and $\hat{u}_0(\varphi-2\theta)$. From this equation, we eliminate $\hat{u}_0(\varphi-2\theta)$ by using the third equation, which operation results in an equation relating $\hat{u}_0(\varphi)$ and $\hat{u}_1(\varphi-3\theta)$, and so on.

As a result, we obtain the equation

$$\hat{u}_0(\varphi)\prod_{k=0}^{n-1}A(\varphi-k\theta) + (-1)^{n+1}\hat{u}_{\nu(n)}(n\theta-\varphi)\prod_{k=1}^{n}A(k\theta-\varphi) = \sum_{j=0}^{n-1}\Big[(-1)^j(\hat{\Phi}_{\nu(j)}(\varphi-j\theta) + \hat{\Phi}_{1-\nu(j)}((j+1)\theta-\varphi))\prod_{k=1}^{j}A(\varphi-k\theta)\prod_{k=j+1}^{n-1}A(k\theta-\varphi)\Big], \tag{15}$$

where $\nu(n) = 0$ for the even n, and $\nu(n) = 1$ for the odd n. In formula (15), it is assumed that, if the lower limit in the product exceeds the upper one, the product will be equal to unity.

Let the wedge angle be

$$\theta = \frac{\pi}{m}. \tag{16}$$

We select $n = m - 1$.

Using the evident property of the coefficient

$$A(\varphi) = A(\pi-\varphi)$$

and performing simple calculations, we represent (15) in the form:

$$K(\varphi)\hat{u}_0(\varphi) + (-1)^{n+1}K(n\theta-\varphi)\hat{u}_{\nu(n)}(n\theta-\varphi) = \sum_{j=0}^{n-1}[(-1)^jV(\varphi-j\theta) \times (\hat{\Phi}_{\nu(j)}(\varphi-j\theta) + \hat{\Phi}_{1-\nu(j)}((j+1)\theta-\varphi))], \tag{17}$$

where

$$K(\varphi) = \prod_{j=1}^{n}[A(j\theta-\varphi)]^{-1},$$

$$V(\varphi) = \prod_{j=0}^{n}[A(\varphi+j\theta)]^{-1}.$$

It is easy to verify that $K(\varphi) = K(-\varphi)$ and $V(\varphi) = V(\theta - \varphi)$. Suppose that the parameter β satisfies the conditions: $0 < \mathrm{Re}\beta < 2\theta$, $\mathrm{Im}\varphi < 0$. Then, the coefficient $K(\varphi)$ will have no poles in the half-band $0 < \mathrm{Re}\varphi < n\theta$, $\mathrm{Im}\varphi > 0$.

Thus, equation (17) represents a problem stated in the same way as the problem represented by equations (10) and (11), except that the function f^- and the parameter Θ are replaced by the combination $K(\varphi)\hat{u}_0(\varphi)$ and the angle $\pi n/m$, respectively. Correspondingly, the solution to this problem is determined by formula (12). The integration path Γ is shown in Fig. 1.

A closer approximation of the solution in the half-band $0 < \mathrm{Re}\varphi < n\theta$, $\mathrm{Im}\varphi > 0$ is given by the formula

$$\hat{u}_0(\varphi) = \frac{1}{2n\theta iK(\varphi)}\int_\Gamma \frac{\sin(\pi\varphi'/n\theta)d\varphi'}{\cos(\pi\varphi'/n\theta) - \cos(\pi\varphi/n\theta)}$$

$$\times \sum_{j=0}^{n-1}((-1)^j V(\varphi' - j\theta)(\hat{\Phi}_{\nu(j)}(\varphi' - j\theta) \tag{18}$$

$$+ \hat{\Phi}_{1-\nu(j)}((j+1)\theta - \varphi'))).$$

We rearrange the right-hand member of this equation by changing the integration variable:

$$\hat{u}_0(\varphi) = \frac{1}{2n\theta iK(\varphi)}$$

$$\times\left(\sum_{j=0}^{[(n+1)/2]-1}\int_{\Gamma-2j\theta}\frac{V(\varphi')\sin(\pi(\varphi'+2j\theta)/n\theta)d\varphi'}{\cos(\pi(\varphi'+2j\theta)/n\theta) - \cos(\pi\varphi/n\theta)}\right.$$

$$\times(\hat{\Phi}_0(\varphi') + \hat{\Phi}_1(\theta - \varphi')) \tag{19}$$

$$+\sum_{j=1}^{[n/2]}\int_{\Gamma+2j\theta-n\theta}\frac{V(\varphi')\sin(\pi(\varphi'-2j\theta)/n\theta)d\varphi'}{\cos(\pi(\varphi'-2j\theta)/n\theta) - \cos(\pi\varphi/n\theta)}$$

$$\left.\times(\hat{\Phi}_0(\varphi') + \hat{\Phi}_1(\theta - \varphi'))\right),$$

where the square brackets indicate the calculation of the integral part. The integration paths can be transformed to the path γ without intersections of the poles of the integrands and without leaving the domains of the decrease at infinity. Taking into account the trigonometric formula

$$\frac{\sin n\psi}{\cos n\psi - \cos n\alpha} = \frac{1}{n}\sum_{j=0}^{n-1}\frac{\sin(\psi - 2\pi j/n)}{\cos(\psi - 2\pi j/n) - \cos\alpha},$$

we bring solution (19) to the form

$$\hat{u}_0(\varphi) = -\frac{1}{2\theta iK(\varphi)}\int_\gamma \frac{V(\varphi')\sin(m\varphi')d\varphi'}{\cos(m\varphi') - \cos(m\varphi)} \tag{20}$$

$$\times(\hat{\Phi}_0(\varphi') + \hat{\Phi}_1(\theta - \varphi')).$$

Using the known representation of the Malyuzhinets function [1]

$$\Psi_{\pi n/4m}(\alpha) = \prod_{k=1}^{m}\prod_{l=1}^{n}\left[\frac{\cos a(k,l)}{\cos((\alpha/n + a(k,l))/2)}\right]^{(-1)^l},$$

$$a(k,l) = \left(\frac{2l-1}{n} - \frac{2k-1}{m}\right),$$

we conclude that, for a wedge angle π/m, where m is even, formula (20) is coincident with solution (9).

Recall that, above, we imposed rather severe conditions on β. If β does not satisfy these conditions, the situation becomes complicated. At arbitrary values of β, the expression $K(\varphi)\hat{u}(\varphi)$ may have simple poles in the half-band $0 < \varphi < n\theta$, $\mathrm{Im}\beta > 0$, which will preclude the direct application of formula (12).

To overcome this difficulty, we can use the following method. We can preset some undetermined values of $\hat{u}(\varphi)$ at the points φ_i corresponding to the poles of $K(\varphi)$ in the given half-band and construct an equation of type (10) for the difference

$$K(\varphi)\hat{u}_0(\varphi) - \sum_i \frac{a_i}{\cos(\pi\varphi/n) - \cos(\pi\varphi_i/n)}.$$

Such an equation allows the direct application of formula (12), but the condition of the decrease at infinity may appear to be insufficient for the calculation of the undetermined values of a_i. This situation has the following explanation. Equation (17) is a consequence of equation (3), while the reverse is not true. At some values of β, a homogeneous equation of type (17) has solutions that are not solutions to the homogeneous equation (3).

To eliminate the ambiguity, we can, for example, complement equation (17) with condition (3) for a finite set of values of the argument represented in the form $\pm\beta + j\theta$, where the absolute value of j does not exceed m. In this case, the condition of the decrease of $\hat{u}(\varphi)$ at infinity is sufficient for the determination of all values of a_i, and the solution again passes into solution (9).

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REFERENCES

1. Malyuzhinets, G.D., Excitation, Reflection, and Radiation of Surface Acoustic Waves by a Wedge with Given Impedances of Its Faces, *Dokl. Akad. Nauk SSSR*, 1958, vol. 121, no. 3, pp. 436–439.
2. Malyuzhinets, G.D., Sound Radiation by Vibrating Faces of an Arbitrary Wedge, parts 1 and 2, *Akust. Zh.*, 1955, vol. 1, no. 2, pp. 144–164; no. 3, pp. 226–234.
3. Tuzhilin, A.A., Theory of Malyuzhinets Functional Equations: I. Homogeneous Functional Equations, General Properties of Solutions, Specific Cases, *Differentsial'nye Uravneniya*, 1970, vol. 6, no. 4, pp. 692–704.
4. Tuzhilin, A.A., Theory of Malyuzhinets Functional Equations: II. Theory of Barnes Infinite Products; General Solutions of Malyuzhinets Homogeneous Functional Equations; Different Representations of the Basic Solution, *Differentsial'nye Uravneniya*, 1970, vol. 6, no. 6, pp. 1048–1063.
5. Tuzhilin, A.A., Theory of Malyuzhinets Functional Equations: III. Inhomogeneous Functional Equations; General Theory, *Differentsial'nye Uravneniya*, 1971, vol. 7, no. 4, pp. 1077–1088.
6. Norris, A.N. and Osipov, A.V., Structural and Acoustical Wave Interaction at a Wedge-Shaped Junction of Fluid-Loaded Plates, *J. Acoust. Soc. Am.*, 1997, vol. 101, no. 2, pp. 867–876.
7. Osipov, A.V. and Norris, A.N., Acoustic Diffraction by a Fluid-Loaded Membrane Corner, *Proc. R. Soc. Lond. A*, 1997, vol. 453, pp. 43–64.
8. Bogy, D.B. and Budaev, B.V., Rayleigh Wave Scattering by a Wedge, *Wave Motion*, 1995, vol. 22, no. 3, pp. 239–257.
9. Shanin, A.V., On Wave Excitation in a Wedge-Shaped Region, *Akust. Zh.*, 1996, vol. 42, no. 5, pp. 696–701 [*Acoust. Phys.*, vol. 42, no. 5, pp. 612–617].
10. Shanin, A.V., Excitation and Scattering of a Wedge Wave in an Obtuse Elastic Wedge Close to 180°, *Akust. Zh.*, 1997, vol. 43, no. 3, pp. 402–408 [*Acoust. Phys.*, vol. 43, no. 3, pp. 344–349].
11. Gautesen, A.K., Scattering of a Rayleigh Wave by an Elastic Wedge, *Wave Motion*, 1987, vol. 9, no. 1, pp. 51–59.
12. Noble, B., *Methods Based on the Wiener–Hopf Technique for the Solution of Partial Differential Equations*, London: Pergamon, 1958.
13. Shanin, A.V., Excitation of Wave Field in a Triangle Area, *International Seminar "Day on Diffraction'97"*, St. Petersburg, 1997.
14. Osipov, A.V., Asymptotic Representation of Acoustic Field in a Narrow Region with Impedance Boundaries, *Akust. Zh.*, 1990, vol. 36, no. 2, pp. 516–522 [*Sov. Phys. Acoust.*, vol. 36, no. 2, pp. 287–290].

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