Any statement below is my private opinion.
This talk contains a lot of controversial point

1st question: Does the Engineering need for Mathematics today?
2nd : If yes, what kind of Mathematics seems to be preferable?

Well-known statement: "Some area of knowledge can be named as a SCIENCE if and only if this area applies the MATHEMATICAL LANGUAGE"

Is this LANGUAGE necessary for any Engineer to communicate with his colleagues? Yes, but that is only a minor constituent.

Basic statement: "Mathematics is much clever, than Mathematician"

It means, that after the scientist or engineer developed new mathematical model and designed corresponding equations, the solution to these equations produces new information unpredictable for the author!

Consider examples

Example 1: Fluid dynamics

To derive equations, one could say only 3 sentences:

1. Conservation of mass takes place in the volume where the motion is localized: $m = \oint \rho \, dV = const$ or in differential form:
$$\frac{\partial \rho}{\partial t} + div \rho \vec{u} = 0$$
2. Conservation of linear momentum takes place in the same volume:
$$\vec{P} = \oint \rho \, \vec{u} \, dV = const$$
or in differential form:
$$\frac{\partial}{\partial t} \rho u_i + \frac{\partial}{\partial x_k} \left(p \delta_{ik} + \rho u_i u_k + \sigma_{ik} \right) = 0$$
3. Viscosity leads to absorption of energy, if velocity depends on coordinates and the motion is different from steady-state rotation of the fluid. From this sentences we derive the viscous stress tensor:
$$\sigma_{ik} = \eta \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial u_l}{\partial x_l} \right) + \varsigma \delta_{ik} \frac{\partial u_l}{\partial x_l}$$

Solutions to these equations describe thousands of new phenomena completely unknown at the derivation of this mathematical model! These equations are used to calculate operation regimes of aircrafts, ships, rockets, industrial flows, weather forecast, etc., etc…

Example 2: Maxwell's equations

$$div\,\vec{D} = 4\pi\,\rho, \quad rot\,\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

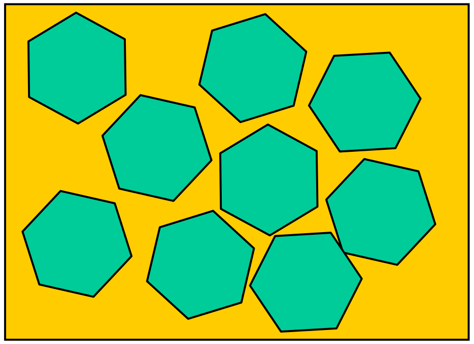
$$div\,\vec{B} = 0, \qquad rot\,\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}.$$

Only a few results of experiments offer the possibility to derive these equations:

1.Coulomb's law 2.Biot-Savart-Laplace law 3.Faraday law

In addition, the existence of displacement current (alternating current can flow through the capacity) and the lack of magnetic charge are taken into account.

Interesting: Maxwell derived these equations using a primitive mechanical model: system of rotating gears with liquid between them



Strikingly! These primitive ideas led to universal equations describing everything: the operation of Radio, TV, Mobile phones, properties of new engineering materials, etc.etc.

How one can study methods of design of new mathematical models and get the unique experience?

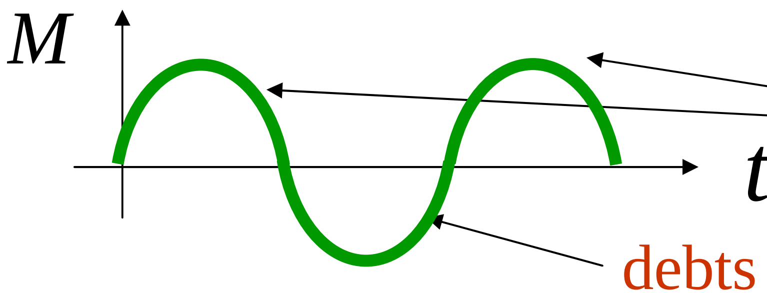
I can demonstrate the simple exercise

A simplest humanitarian example

Philosophy of a poor painter: The more money, the less happiness

Solution to this system:

$$\frac{dH}{dt} = -aM, \qquad \frac{dM}{dt} = +bH$$

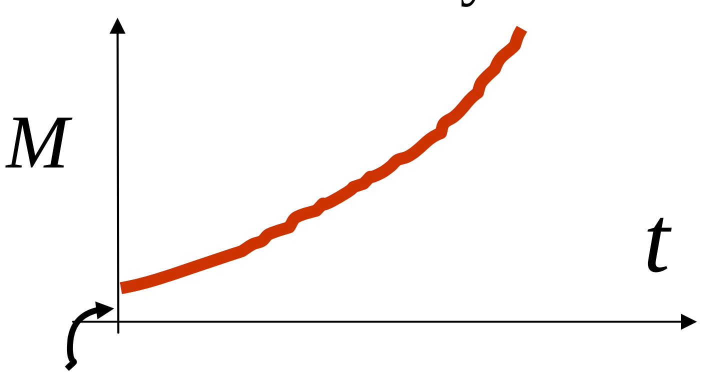


Periods of **Comfortable life** alternate with **distress**

Philosophy of a businessman: More money - more happiness

Solution to this system:

$$\frac{dH}{dt} = +aM, \qquad \frac{dM}{dt} = +bH$$



laid-down capital

Amount of money increases exponentially!

Simple example of design of mathematical model (REAL Mathematical Modeling)

Simplest problem of demography: variation of male and female population with account of birth and death only.

$$\frac{dM}{dt} = +bMF - \frac{M}{t_M} \qquad \frac{dF}{dt} = +gMF - \frac{M}{t_F}$$

Temporal change in number of Males

Increases with increase in frequency of intimate relations between M and F

Decreases because of mortality

The same ideas lead to the 2nd equation for Females

Notations: M – number of Males, F – number of Females, t_M , t_F -corresponding life intervals, $b,\ g = (1-b)$ -probabilities of birth of boys and girls. The simplest solution is the stationary one, when M and F are constant. Requirement M=F leads to simple formula:

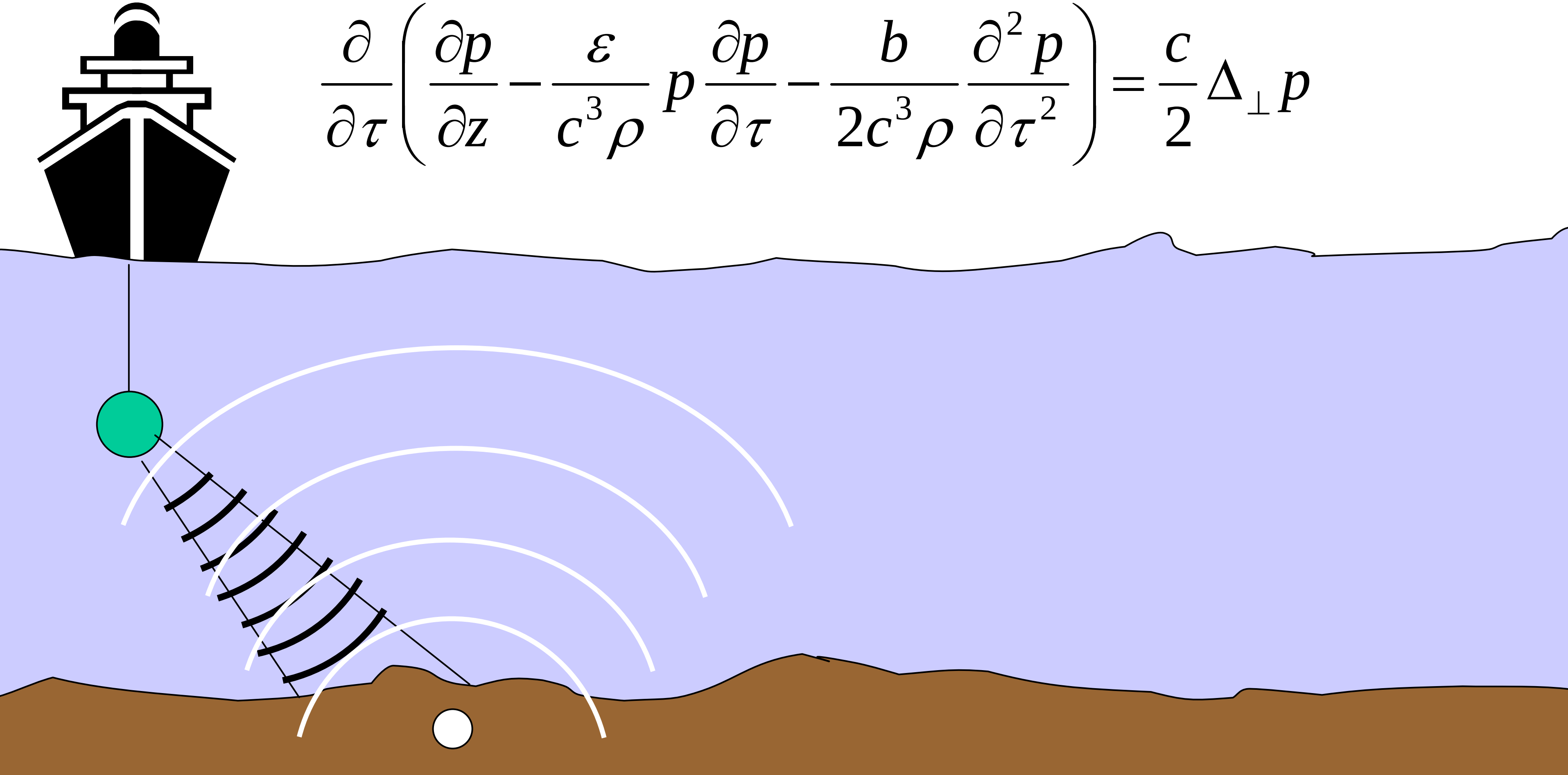
$$b = \frac{t_F}{t_F + t_M}$$

If life interval for women is 70 years and for men 50 years (Russia), we calculate b =7/12. It means, that from among 12 of born babies, 7 must be boys and 5 must be girls

A little of my own experience in real mathematical modeling

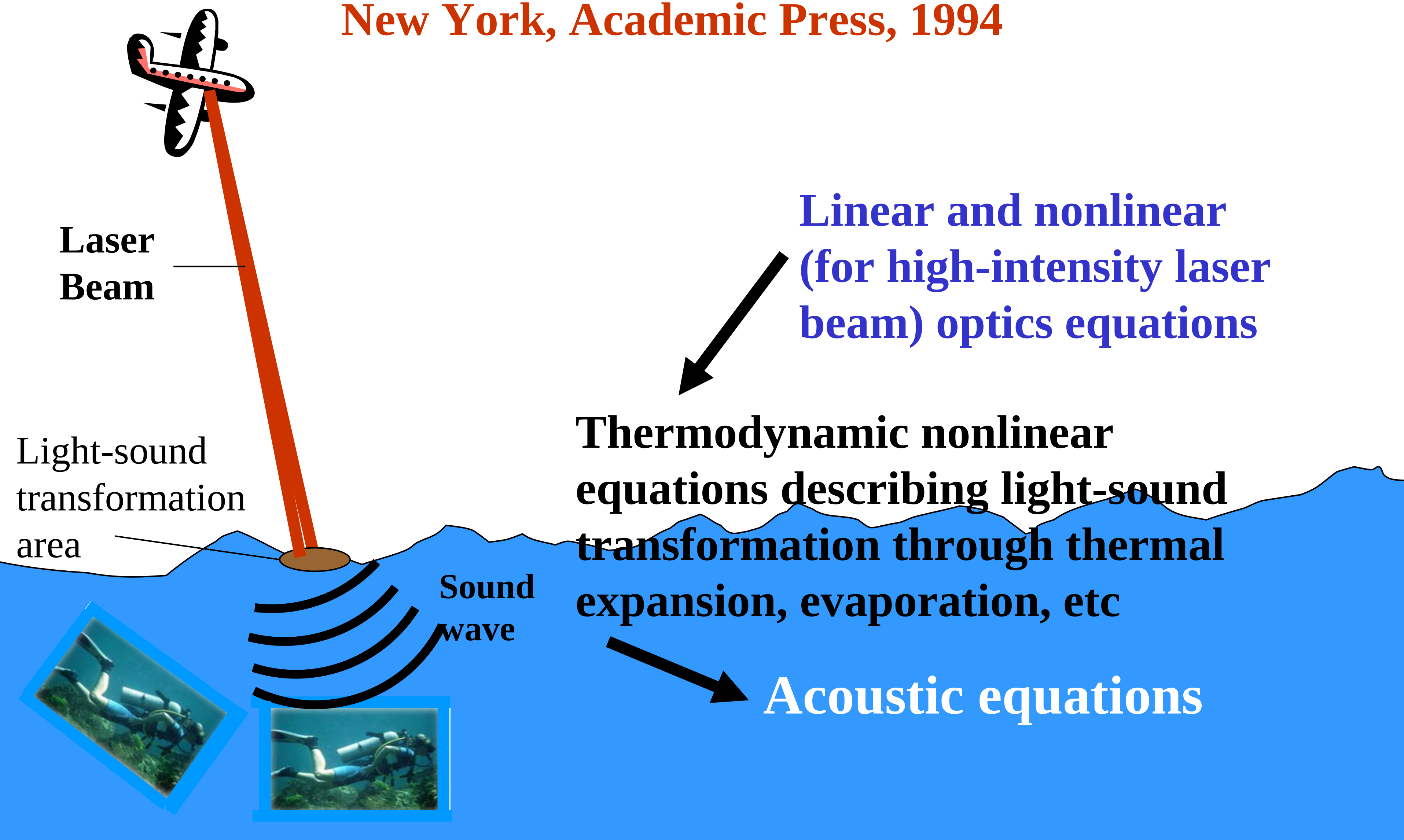
1. Parametric underwater sonar (echo depth-sounding, fishery, mine location in marine sediments) was produced industrially after the engineering design method based on nonlinear acoustic equations was developed (See details in the book: B.K.Novikov, O.V.Rudenko, V.I.Timoshenko "Nonlinear Underwater Acoustics", American Inst.of Physics, 1987 – translated from Russian ed., 1981)

$$\frac{\partial}{\partial \tau}\left(\frac{\partial p}{\partial z} - \frac{\varepsilon}{c^3 \rho} p \frac{\partial p}{\partial \tau} - \frac{b}{2c^3 \rho} \frac{\partial^2 p}{\partial \tau^2}\right) = \frac{c}{2} \Delta_{\perp} p$$

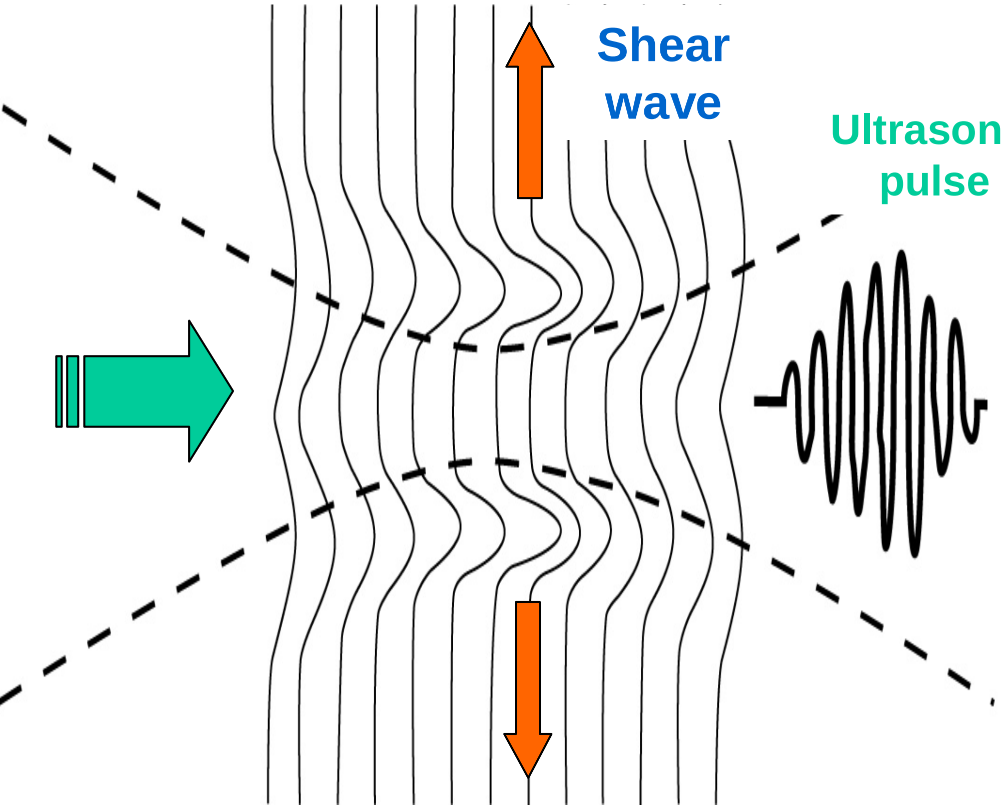


Signal transmission from air to underwater

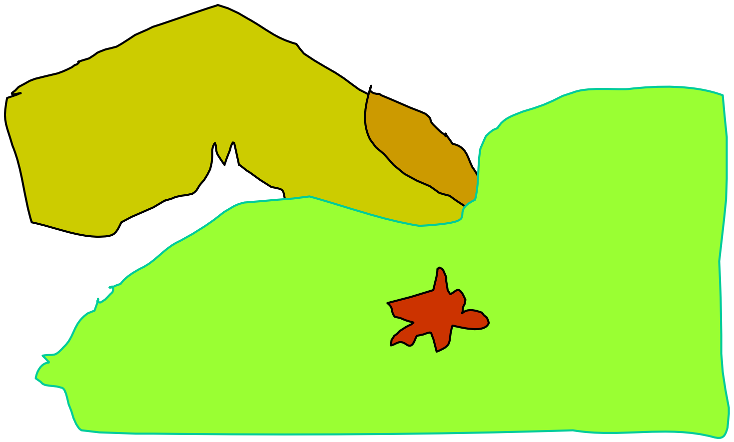
V.E.Gusev, A.A.Karabutov. Laser optoacoustics. New York, Academic Press, 1994



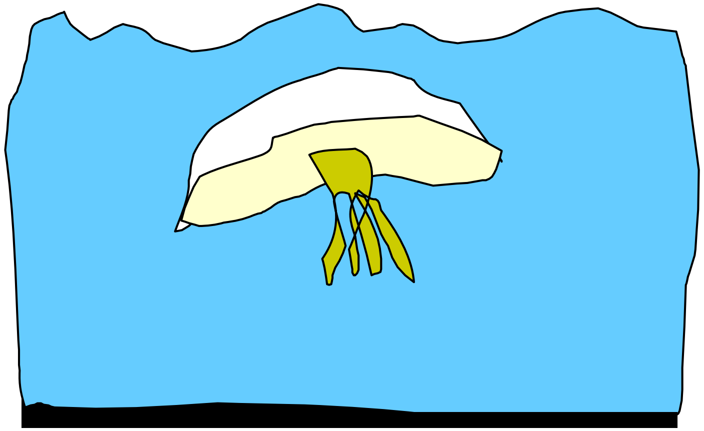
Modulated radiation pressure for shear wave excitation



Palpation to detect Tumor: is that region of increased density?



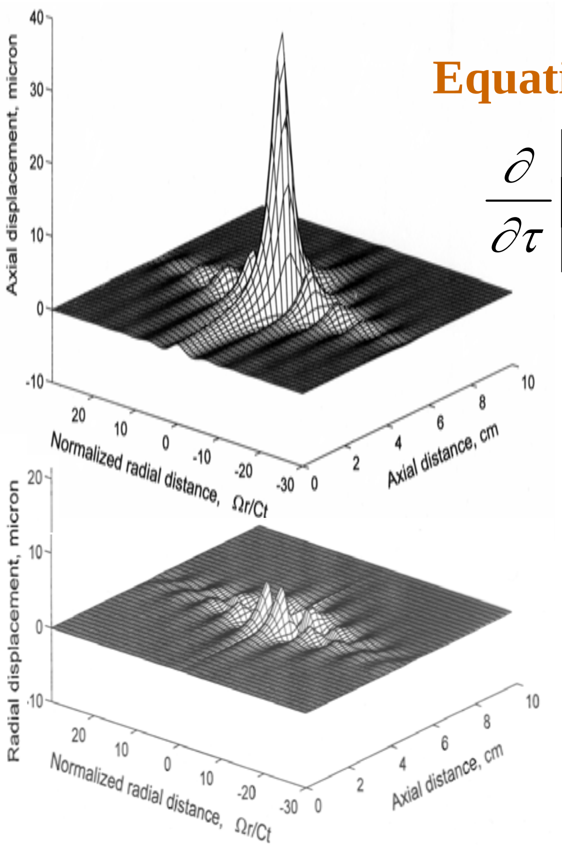
No, density of tumor is the same as density of surrounding tissue



Application to medical diagnostics: Shear Wave Elasticity Imaging

Sarvazyan and Rudenko. US Patent 5,810,731 (Sept.22, 1998)

Mathematical model for «Shear Wave Elasticity Imaging»



Equation for nonlinear ultrasonic beam in tissue

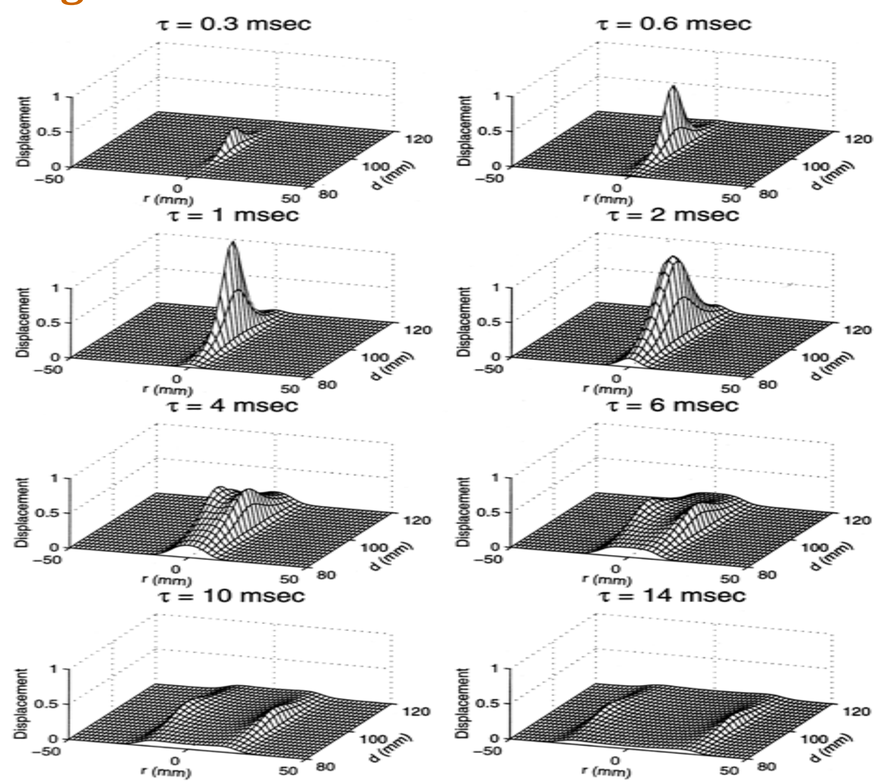
$$\frac{\partial}{\partial \tau}\left[\frac{\partial p}{\partial x}-\frac{\varepsilon}{c^3\rho}\,p\,\frac{\partial p}{\partial \tau}-\gamma\,\frac{\partial^2}{\partial \tau^2}\int_0^{\infty} \mathrm{K}(\xi)\,p(x,\tau-\xi)\,d\xi\right]=\frac{c}{2}\Delta_{\perp} p$$

Boundary condition on focusing transducer

$$p(x=0,r,t)=p_0\Phi\left(\frac{r^2}{a^2}\right)\varphi(\Omega t)\cdot\sin\omega\left(t+\frac{r^2}{2cd}\right)$$

Averaging solution gives Radiation Pressure

$$F_x=\frac{b}{c^3\rho}\overline{\left(\frac{\partial p}{\partial \tau}\right)^2}$$



This pressure is pulsating because US is modulated in kHz frequencies. This vibration excites the shear wave in tissue

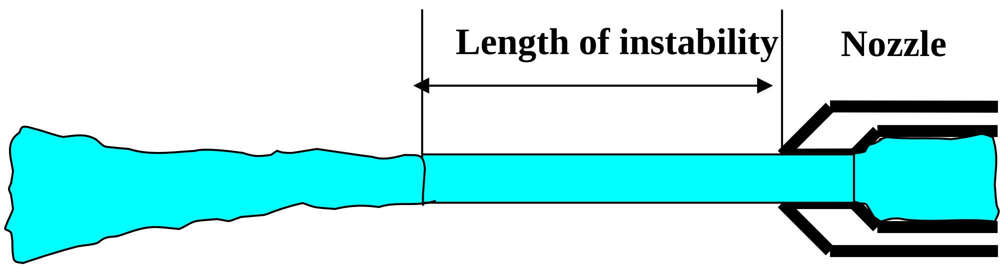
$$\frac{\partial^2 s_x}{\partial t^2}-(c_t^{\,2}+\nu\frac{\partial}{\partial t})\Delta_{\perp} s_x=F_x$$

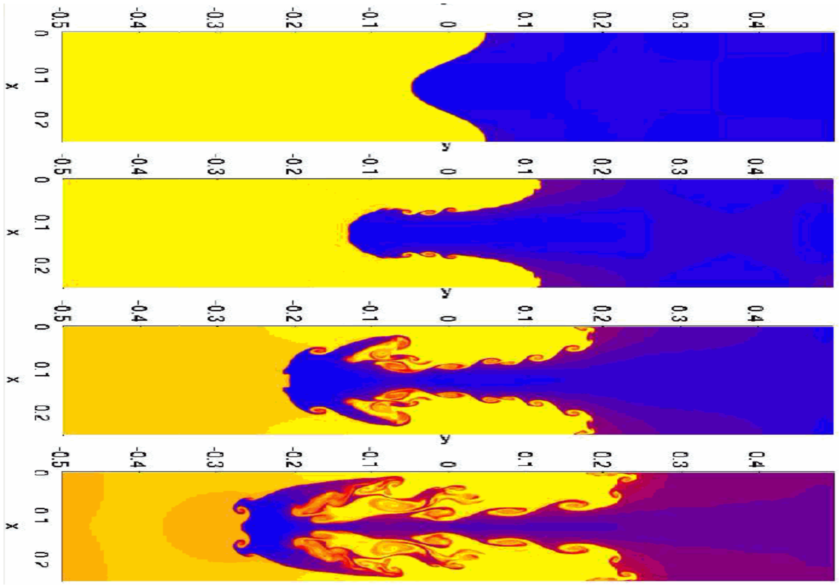
Studies in Fluid Dynamics of water jet are connected with two main problems:

1. Flows in contracting nozzle forming high-speed jet. 2. Instability of water jet in air. Results are briefly described below.

Instability leads to breakdown of jet and to catastrophic decrease in its cutting ability.

Several physical factors were taken into account: (i) capillary forces at water free surface determined by both surface tension of pure water and polymer fibers added artificially to improve stability; (ii) aerodynamic forces appearing at the streamlining of jet irregularities.





It was shown, that jet velocities around sound speed in air (330 m/s) are very undesirable, because instability increases explosively due to the wave resonance phenomenon. Next undesirable factor is the presence of air bubbles in liquid.

For instability length the solution is derived:

$$L_{ins}^{-1} = \frac{1}{U}\sqrt{\frac{I_1(kR)}{\rho_L I_0(kR)}}\sqrt[4]{(a-c)^2+b^2}\,\sin\left(\frac{1}{2}\arctan\frac{b}{a-c}\right),$$

$$a = \rho_A k^2 \frac{U^2}{\beta}\frac{J_0J_1+Y_0Y_1}{J_1^2+Y_1^2}, \quad b = \rho_A k^2\frac{U^2}{\beta}\frac{2}{\pi k R}\frac{1}{J_1^2+Y_1^2},$$

$$c = \frac{\sigma k}{R^2}\left(k^2R^2-1\right)$$

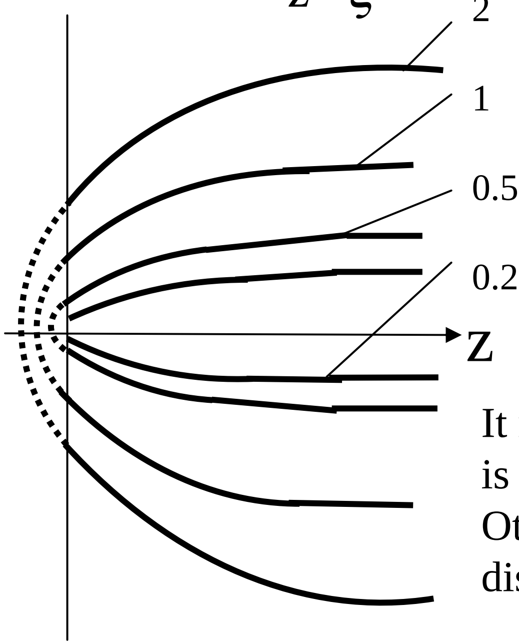
To form the high-speed jet having velocities in air from 200 to 900 m/s, it is necessary to accelerate liquid in contracting nozzle by difference in pressure between input and output cross-sections. This fast (nonlinear) flow is described by equations:

$$(\vec{u}\nabla)\vec{u} + \nu\,\Delta\vec{u} = -\nabla\left(p/\rho_0\right), \quad div\,\vec{u} = 0$$

Exact solution depending on 3 constants was derived:

$$u_z = \frac{1}{z}W, \quad W = \frac{D}{2}\frac{\sqrt{a^2+\xi^2}}{1+\xi^2}\left[\sqrt{\frac{a^2+\xi^2}{2-a^2+\xi^2+C_3\sqrt{1+\xi^2}}} + \sqrt{\frac{2-a^2+\xi^2+C_3\sqrt{1+\xi^2}}{a^2+\xi^2}}\right]$$

$$u_r = \frac{1}{z}\,\frac{W}{\xi}\,\frac{2\left(a^2-2\right)\sqrt{1+\xi^2} - C_3\left(2+\xi^2\right)}{2\sqrt{1+\xi^2}+C_3}, \quad \xi = \frac{r}{z}$$



$$\kappa_{1,2} = \frac{4QL^2}{\pi\sqrt{p_0\rho_0}\left(r_2^2-r_1^2\right)^2}\,\frac{\ln^{1/2}\left(1+\frac{r_2^2-r_1^2}{4L^2}\frac{L}{z_{1,2}}\right)}{\sqrt{1+\frac{4L^2}{r_2^2-r_1^2}\frac{z_{1,2}}{L}}}$$

This exact solution gives optimum shape of nozzle and formulas connecting main geometrical parameters of nozzle and water flow:

$$\frac{Q}{\pi\sqrt{p_0\rho_0}} = r_1^2 - \left(\frac{r_2^2-r_1^2}{2L}\right)^2$$

An important equation exists connecting the total mass stream (outlet discharge) and geometrical parameters of the nozzle.

Strong restriction on dimensions: $2Lr_1 > r_2^2 - r_1^2$

It means that nozzle length must be large enough. For example, if output radius is 1mm, and input radius is 1 cm, the nozzle length must be not less than 5 cm. Otherwise the stream considered above cannot be realized. This restriction was discovered experimentally by Water Jet Sweden and explained by this theory.

MATHEMATICS PHYSICS ENGINEERING

Real Mathematical Modeling:

Level 1: Designing of New equations to describe processes in Physics and Engineering

Level 2: **Exact Solution:** the best way to extract information from the mathematical model

Level 3: **Approximate calculations and asymptotic methods** – are used if mathematicians cannot find the exact solution

Level 4: **Computer simulation** (incorrect interpretation of “Mathematical Modeling”) is used to solve complicated problems after main phenomena are studied at Level 2 and Level 3. Direct computer analysis of existing Models using unknown software leads often to principal mistakes