A generalization of the separation of variables method for some 2D diffraction problems

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Abstract

A new method is proposed for solving diffraction problems having piecewise linear ideal boundaries. According to this method, the initial Helmholtz equation is replaced by a system of matrix equations of order 1. The coefficients of these equations are rational matrix functions of the coordinates. The properties of the coordinate matrix system are close to that of the ordinary differential equation, therefore the new method can be treated as a generalization of the separation of variables.

1 Introduction

Separation of variables (SV) is the most powerful technique for finding exact solutions of the problems related to partial differential equations. However, this method has some weak points. Two of them are the following. First, this method is applicable to a very restricted set of geometries. All these geometries, say for the Helmholtz equation, are classified by group theory analysis; the detailed discussion of this matter can be found, for example, in [1]. Second, even a simple diffraction problem leads to a complicated Fourier series in special functions. As a rule, the structure of the field cannot be easily found from the series. A special technique, such as Watson's transform, is required to study the exact solution obtained using SV. Sadly, such a transform is not known for many interesting cases.

In the present paper we propose a technique that can help to overcome the disadvantages mentioned above, i.e. it is applicable to a wider class of problems and the solution is not represented as a Fourier series. The main idea of the method is to reduce the diffraction problem, which involves the Helmholtz equation, boundary, edge and radiation conditions, to an ordinary differential equation (ODE) of a rather simple form. The coefficients of the equation are rational functions of the coordinates.

Here we describe the main idea of the SV generalization. Consider the simplest case of SV: the wave field in a rectangular area. Consider the Helmholtz equation

$$\Delta u + k_0^2 u = 0 \tag{1.1}$$

in a rectangle $0 < x < L$, $0 < y < M$ with Dirichlet conditions at the boundaries, $u = 0$. The solution is the product

$$u = \sin kx \sin \gamma y, \qquad k = \frac{\pi m}{L}, \qquad \gamma = \sqrt{k_0^2 - k^2} = \frac{\pi n}{M}, \qquad m, n \in Z. \tag{1.2}$$

Construct the vector $\mathbf{U} = (u^1, u^2, u^3, u^4)^T$ of dimension 4 with the following components:

$$u^1 = u, \qquad u^2 = \frac{\partial u}{\partial x}, \qquad u^3 = \frac{\partial u}{\partial y}, \qquad u^4 = \frac{\partial^2 u}{\partial x \partial y}.$$

Note that all components of the vector are solutions of the Helmholtz equation in the rectangle, but the boundary conditions for these solutions should be chosen differently.

One can check, that the vector $\mathbf{U}$ obeys the differential equations

$$\frac{\partial \mathbf{U}}{\partial x} = \mathbf{X}\mathbf{U}, \qquad \frac{\partial \mathbf{U}}{\partial y} = \mathbf{Y}\mathbf{U}, \tag{1.3}$$

with the matrix coefficients

$$\mathbf{X} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -k^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -k^2 & 0 \end{pmatrix}, \qquad \mathbf{Y} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\gamma^2 & 0 & 0 & 0 \\ 0 & -\gamma^2 & 0 & 0 \end{pmatrix}. \tag{1.4}$$

The situation is similar for each classical SV. Let η and ξ be the coordinates, in which the variables are separated. Then an eigenfunction can be written as a product $u = T(\eta)S(\xi)$, and the functions T and S obey differential equations of order 2:

$$T''(\eta) - f_1(\eta)T'(\eta) - g_1(\eta)T(\eta) = 0,$$
$$S''(\xi) - f_2(\xi)S'(\xi) - g_2(\xi)S(\xi) = 0,$$

One can see that the following equations are valid (we denote the derivatives by the indices here):

$$\frac{\partial}{\partial \eta} \begin{pmatrix} u \\ u_{,\eta} \\ u_{,\xi} \\ u_{,\eta\xi} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ g_1(\eta) & f_1(\eta) & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g_1(\eta) & f_1(\eta) \end{pmatrix} \begin{pmatrix} u \\ u_{,\eta} \\ u_{,\xi} \\ u_{,\eta\xi} \end{pmatrix}, \tag{1.5}$$

$$\frac{\partial}{\partial \xi} \begin{pmatrix} u \\ u_{,\eta} \\ u_{,\xi} \\ u_{,\eta\xi} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ g_2(\xi) & 0 & f_2(\xi) & 0 \\ 0 & g_2(\xi) & 0 & f_2(\xi) \end{pmatrix} \begin{pmatrix} u \\ u_{,\eta} \\ u_{,\xi} \\ u_{,\eta\xi} \end{pmatrix}. \tag{1.6}$$

The equations (1.3) will now be called the *coordinate equations.* These equations formally are in partial derivatives, but their properties are similar to that of an ordinary differential equation. Namely, the solution of a partial differential equation can be found when its values are known on a contour in the (x, y)-plane. Conversely, the solution of the system (1.3) is specified by the value of the vector $\mathbf{U}$ at just one arbitrary point P. Any other point P' can be connected with P by a line, and an ordinary differential equation describing the behaviour of $\mathbf{U}$ on this line can be constructed as a restriction of (1.3) on this line. The value $\mathbf{U}(P)$ should be used as the initial conditions for this equation.

One can see that the coefficients of the equations (1.5) and (1.6) have a very strict structure. Some coefficients should be equal to zero, and the other ones should depend on a single variable. The idea of generalization these equations is by allowing the unknown vector $\mathbf{U}$ and the coefficient matrices to have a more flexible structure. We demand only that the first component of the vector is the solution of the given diffraction problem, all other components can be chosen arbitrarily. The vector can have an arbitrary dimension, and the elements of the coefficient matrices can be arbitrary functions of the coordinates. Obviously, the structure of the coefficents should be much simpler than the structure of the solutions, otherwise the equations are senseless.

Surprisingly, equations of the form (1.3) with relatively simple coefficients can be written for a wide class of diffraction problems.

Of course, some price is paid for this generalization of the SV method. While the order of the ODEs in the classical SV applied to the Helmholtz equation is normally equal to 2, the order of the ODEs emerging in our method is usually higher than 2. The more complicated scatterer leads to equation of higher order. The result is a situation, in which the analytical solution of the ODE is not known. However, the equations obtained here belong to a well-studied class: they are confluent Fuchsian. A powerful theory has been developed for these equations, so the situation is not completely hopeless.

Our paper has a strong relation to the works [2], [3], [4] devoted to the strip problem. The ODEs obtained in these works for the field on the scatterer or for the far-field diagram can be found as particular cases of the coordinate equations obtained here. The procedure proposed below is close to that of [3]. We should mention here also the work [5], where the structure of the eigenfunction series for the several strips problem was constructed. However, the relation of these series to our method still remains questionable.

The paper is organized as follows. In Section 2 the coordinate equations are derived for the problem of diffraction of a plane wave by two Dirichlet or Neumann strips. The coefficients and the initial conditions are known up to several numerical constants, which should be found either by solving the eigenvalue problem or using some other technique. In Section 3 some special issues related to the coordinate equations are discussed. The solvability of the equations and obeying the Helmholtz equation and the boundary conditions by the solutions are these issues. The equations for the directivities are also derived in this section. In Section 4 we represent the analytical solution of the coordinate equations for the case of diffraction by a half-plane and the numerical solution of the coordinate equation for the case of two strips.

2 Derivation of the coordinate equations for the problem of diffraction by two strips

2.1 Problem formulation

Let the Helmholtz equation (1.1) be fulfilled in the (x, y)-plane. The time dependence is assumed to have the form $e^{-\mathrm{i}\omega t}$ and is omitted everywhere. The incident plane wave

$$u_{\mathrm{in}} = e^{-\mathrm{i}k_* x - \mathrm{i}\sqrt{k_0^2 - k_*^2}\, y}, \qquad k_* = k_0 \cos\psi$$

(ψ is the angle of incidence) illuminates the scatterer which is a set of two strips (see Fig. 1) with Dirichlet boundary conditions $u^{\mathrm{d}} = 0$.

The problem is effectively two-dimensional. The (x, y)-cross-section of the strips is the set of the segments (a_1, a_2), (a_3, a_4) of the x-axis. All the techniques presented below can be obviously generalized to the case of an arbitrary number of the strips located in one plane and bearing the Dirichlet boundary conditions, The case of 2 strips is chosen just because it is the simplest case, for which the classical SV is not known. Note that our method gives nontrivial results even for the cases of one strip and a half-plane, for which the classical SV is applicable.

The total field u^{d} can be split into the incident and the scattered field:

$$u^{\mathrm{d}} = u_{\mathrm{in}} + u^{\mathrm{d}}_{\mathrm{sc}}.$$

The scattered field is symmetrical. The boundary conditions for $u^{\mathrm{d}}_{\mathrm{sc}}$ are as follows.

$$u^{\mathrm{d}}_{\mathrm{sc}}(x, \pm 0) = -e^{-\mathrm{i}k_* x} \qquad \text{for} \qquad x \in (a_1, a_2) \cup (a_3, a_4), \tag{2.1}$$

$$\partial_y u^{\rm d}_{\rm sc}(x,0) = 0 \qquad \text{for} \qquad x \in (-\infty, a_1) \cup (a_2, a_3) \cup (a_4, \infty). \tag{2.2}$$

We assume that the radiation condition is fulfilled for the scattered field. Moreover, Meixner's conditions are fulfilled in the vicinity of the *edges* of the scatterer, i.e. near the points $(a_m, 0)$. In our case Meixner's conditions conditions demand that the field has the asymptotics

$$u \sim A + B(r^{1/2}) + O(r)$$

near the edges.

We formulate the problem of the diffraction by a set of Neumann strips as well. Let the geometry of the strips be the same as for the Dirichlet problem. Let the Neumann boundary conditions be fulfilled by the field $u^{\rm n}$ on the strips:

$$\partial_y u^{\rm n} = 0 \qquad \text{for} \quad y = \pm 0, \quad x \in (a_1, a_2) \cup (a_3, a_4).$$

The total field, again, can be represented as the sum

$$u^{\rm n} = u_{\rm in} + u^{\rm n}_{\rm sc},$$

$u^{\rm n}_{\rm sc}$ is the scattered field, which is antisymmetric and satisfies the following boundary conditions:

$$\partial_y u^{\rm n}_{\rm sc}(x, \pm 0) = {\rm i}\sqrt{k_0^2 - k_*^2} e^{-{\rm i}k_* x} \qquad \text{for} \qquad x \in (a_1, a_2) \cup (a_3, a_4), \tag{2.3}$$

$$u^{\rm n}_{\rm sc}(x, 0) = 0 \qquad \text{for} \qquad x \in (-\infty, a_1) \cup (a_2, a_3) \cup (a_4, \infty). \tag{2.4}$$

The radiation and the edge conditions are fulfilled by $u^{\rm n}_{\rm sc}$.

2.2 Auxiliary solutions for the strip and the slit problem

The derivation of the coordinate equations for $u^{\rm d,n}_{\rm sc}$ is performed in two steps. Firstly, the auxiliary (oversingular) functions are introduced and the coordinate equations for them are derived. Secondly, the coordinate equations for the $u^{\rm d,n}_{\rm sc}$ are derived using the equations for the auxiliary functions.

Introduce the local cylindrical coordinates (ρ_m, θ_m) near the edges of the scatterer (see Fig. 2).

Consider the diffraction problem with the same Dirichlet strips as described above, but with the excitation by a line source rather than by a plane wave. Let the source be located near one of the edges $(a_m, 0)$. Namely, let v^m, $m = 1 \dots 4$ be the limit of the solution of the inhomogeneous Helmholtz equation

$$\Delta v^m + k_0^2 v^m = \pi^{1/2} \epsilon^{-3/2} \delta(\rho_m - \epsilon) \delta(\theta_m - \pi), \tag{2.5}$$

Here δ is the Dirac's delta-function; $\epsilon \to 0$. The strength of the source is equal to $\epsilon^{-1/2}$ Note that the dependence of strength of the source on ϵ is chosen such that there exists the non-zero limit of the solution. The boundary condition for v^m have the form

$$v^m(x, \pm 0) = 0 \qquad \text{for} \qquad x \in (a_1, a_2) \cup (a_3, a_4), \tag{2.6}$$

Obviously, the function v^m is symmetrical, hence the following boundary condition is valid:

$$\partial_y v^m(x, 0) = 0 \qquad \text{for} \qquad x \in (-\infty, a_1) \cup (a_2, a_3) \cup (a_4, \infty). \tag{2.7}$$

Also, the radiation condition is fulfilled by v^m.

Meixner's conditions are assumed to be satisfied for each non-zero ϵ, but the behaviour of the limiting function v^m near the edge $(a_m, 0)$ appears to violate Meixner's condition just because the presence of the source near this edge. This behaviour can be studied by the standard technique. Taking the limit, we obtain that the function behaves like $v^m \sim \rho_m^{-1/2}$. Therefore, instead of taking the limit of the line-source problem, one can find v^m as the solution having the *oversingular* behaviour near the edge a_m.

In more details, the asymptotics of the solutions v^m near the edges $(a_n, 0)$ have the form

$$v^m(\rho_n, \theta_n) = -\frac{\delta_{m,n}}{\sqrt{\pi}} \rho_n^{-1/2} \sin \frac{\theta_n}{2} + \frac{2C_n^m}{\sqrt{\pi}} \rho_n^{1/2} \sin \frac{\theta_n}{2} + O(\rho_n^{3/2}), \tag{2.8}$$

where C_n^m are some unknown coefficients; $\delta_{m,n}$ is the Kronecker's delta.

We also introduce the auxiliary oversingular functions w^m, $m = 1, \dots 4$ for the Newmann strips problem.

The boundary conditions have the form

$$\partial_y w^m(x, \pm 0) = 0 \qquad \text{for} \qquad x \in (a_1, a_2) \cup (a_3, a_4), \tag{2.9}$$

$$w^m(x, 0) = 0 \qquad \text{for} \qquad x \in (-\infty, a_1) \cup (a_2, a_3) \cup (a_4, \infty). \tag{2.10}$$

The radiation condition is fulfilled. The Meixner's conditions are satisfied for each non-zero ϵ. The functions w^m are characterised by the oversingular terms in their asymptotics:

$$w^m(\rho_n, \theta_n) = -\frac{\delta_{m,n}}{\sqrt{\pi}} \rho_n^{-1/2} \cos \frac{\theta_n}{2} + \frac{2E_n^m}{\sqrt{\pi}} \rho_n^{1/2} \cos \frac{\theta_n}{2} + O(\rho_n^{3/2}), \tag{2.11}$$

E_n^m are some constants.

The constants C_n^m and E_n^m play an important role in the subsequent derivation of the coordinate equations. Namely, they are the numerical parameters of the coefficients $\mathbf{X}$ and $\mathbf{Y}$. At this stage these parameters are unknown, but we assume that they can be calculated by some technique.

2.3 An important note

We suppose that k_0 belongs neither to the spectrum of the Dirichlet nor of the Neumann strips problem. It means that the functions $u_{\mathrm{sc}}^{\mathrm{d,n}}$ are defined uniqiely and the following statement is true. If some function $u(x, y)$ obeys the homogeneous Helmholtz equation, radiation condition at infinity, homogeneous boundary conditions (2.4), (2.3) or (2.9), (2.10) on the x-axis and Meixner's conditions at the edges, then $u \equiv 0$. This feature is very important for derivation of the coordinate equations. If k_0 belongs to the spectrum, the method should be modified.

Note that the auxiliary solutions v^m and w^m are allowed to be non-zero only because they violate Meixner's condition at one of the edges.

2.4 Derivation of the coordinate equations for the auxiliary functions

Construct the vector of the unknowns

$$\mathbf{U} = (v^1, v^2, v^3, v^4, w^1, w^2, w^3, w^4)^T. \tag{2.12}$$

Our purpose here is to construct the equations of the form (1.3) for this vector. We find it convenient also to introduce the vectors of dimension 4:

$$\mathbf{V} = (v^1, v^2, v^3, v^4)^T, \qquad \mathbf{W} = (w^1, w^2, w^3, w^4)^T.$$

Construct the derivative

$$v^* \equiv \frac{\partial v^m}{\partial \theta_m} = \left[(x - a_m)\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \right] v^m,$$

where θ_m is treated as a global cylindrical coordinate. Obviously, v^* obeys the homogeneous Helmholtz equation (1.1) and the radiation condition. Moreover, detailed study shows that v^* satisfies the boundary conditions (2.9) and (2.10) (note that v^m is antisymmetrical and smooth enough on the strips, and that $\partial_y^2 v^m = -(\partial_x^2 + k_0^2) v^m$).

Meixner's conditions are violated by v^*. Using the asymptotics (2.8) one can obtain asymptotic estimations for v^* near the edge a_n:

$$v^m = \left(\frac{(-1)^m \delta_{m,n}}{2} + (a_n - a_m) C_n^m \right) \frac{1}{\sqrt{\pi}} \rho_n^{-1/2} \cos\frac{\theta_n}{2} + O(\rho_n^{1/2}). \tag{2.13}$$

Using these asymptotics, one can construct the combination

$$v^{**} = v^* + \frac{(-1)^m}{2} w^m + \sum_{n=1}^{4} (a_n - a_m) C_n^m w^n,$$

which satisfies the Meixners's conditions. Besides, v^{**} satisfies the Helmholtz equation, radiation condition and the boundary conditions (2.9), (2.10). Due to Subsection 2.3, $v^{**} \equiv 0$. Therefore,

$$\left[(x - a_m)\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right] v^m = \mathbf{R}^m \mathbf{W} \equiv \sum_{n=1}^{4} R_n^m w^n, \qquad m = 1 \dots 4, \tag{2.14}$$

where $\mathbf{R}^m$ is the string with the elements

$$R_n^m = \frac{(-1)^{m-1}\delta_{m,n}}{2} - (a_n - a_m)C_n^m, \qquad n = 1 \dots 4. \tag{2.15}$$

Thus, one equation expressing the spatial derivatives of v^m through the components of the vector $\mathbf{U}$ is constructed. We need more such equations to obtain the separate expressions for the derivatives. For this we should find several combinations containing the derivatives of v^m and w^m with respect to x and y and having the singularities at $(a_n, 0)$ not stronger than $\rho_n^{-1/2}$. One can check directly that the combinations

$$\left[(x - a_m)\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right] w^m, \qquad \frac{\partial v^m}{\partial y} + (-1)^m \frac{\partial w^m}{\partial x}, \qquad \frac{\partial w^m}{\partial y} - (-1)^m \frac{\partial v^m}{\partial x}$$

satisfy this condition. Using the technique described above, i.e. studying the asymptotics of these combinations, subtracting the appropriate sums of v^n or w^n and taking into account the uniqueness of the solution, we obtain the following representations:

$$\left[(x - a_m)\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right] w^m = \mathbf{Q}^m \mathbf{V} \equiv \sum_{n=1}^{4} Q_n^m v^n, \qquad m = 1 \dots 4, \tag{2.16}$$

$$\frac{\partial v^m}{\partial y} + (-1)^m \frac{\partial w^m}{\partial x} = \mathbf{S}^m \mathbf{W} \equiv \sum_{n=1}^{4} S_n^m w^n, \qquad m = 1 \dots 4, \tag{2.17}$$

$$\frac{\partial w^m}{\partial y} - (-1)^m \frac{\partial v^m}{\partial x} = \mathbf{T}^m \mathbf{V} \equiv \sum_{n=1}^{4} T_n^m v^n, \qquad m = 1 \dots 4, \tag{2.18}$$

where $\mathbf{Q}^m$, $\mathbf{S}^m$ and $\mathbf{T}^m$ are the raws of dimension 4:

$$Q_n^m = \frac{(-1)^m \delta_{m,n}}{2} - (a_n - a_m)E_n^m, \tag{2.19}$$

$$S_n^m = -C_n^m - (-1)^{m+n} E_n^m, \tag{2.20}$$

$$T_n^m = -E_n^m - (-1)^{m+n} C_n^m \tag{2.21}$$

for $n = 1 \ldots 4$.

The equations (2.14), (2.16), (2.17), (2.18) can be solved with repect to the x- and y-derivatives of v^m and w^m for each m. The result can be written in the form of the coordinate equations

$$\frac{\partial \mathbf{U}}{\partial x} = \mathbf{X}\mathbf{U}, \qquad \frac{\partial \mathbf{U}}{\partial y} = \mathbf{Y}\mathbf{U}, \tag{2.22}$$

where $\mathbf{X}$ and $\mathbf{Y}$ are the block matrices

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1^1 & \mathbf{X}_2^1 \\ \mathbf{X}_1^2 & \mathbf{X}_2^2 \end{pmatrix}, \qquad \mathbf{Y} = \begin{pmatrix} \mathbf{Y}_1^1 & \mathbf{Y}_2^1 \\ \mathbf{Y}_1^2 & \mathbf{Y}_2^2 \end{pmatrix}, \tag{2.23}$$

The elements of the matrices $\mathbf{X}_j^i$ and $\mathbf{Y}_j^i$ are given by the following formulae:

$$(X_1^1)_n^m = \frac{(-1)^{m-1}(x-a_m)(Q_n^m - (x-a_m)T_n^m)}{(x-a_m)^2+y^2}, \quad (X_2^1)_n^m = -\frac{y(R_n^m - (x-a_m)S_n^m)}{(x-a_m)^2+y^2},$$
$$(X_1^2)_n^m = -\frac{y(Q_n^m - (x-a_m)T_n^m)}{(x-a_m)^2+y^2}, \quad (X_2^2)_n^m = \frac{(-1)^m(x-a_m)(R_n^m - (x-a_m)S_n^m)}{(x-a_m)^2+y^2},$$
$$(Y_1^1)_n^m = \frac{(-1)^{m-1}y(Q_n^m - (x-a_m)T_n^m)}{(x-a_m)^2+y^2}, \quad (Y_2^1)_n^m = \frac{(x-a_m)R_n^m + y^2 S_n^m}{(x-a_m)^2+y^2},$$
$$(Y_1^2)_n^m = \frac{(x-a_m)Q_n^m + y^2 T_n^m}{(x-a_m)^2+y^2}, \quad (Y_2^2)_n^m = \frac{(-1)^m y(R_n^m - (x-a_m)S_n^m)}{(x-a_m)^2+y^2}.$$

Thus, the coordinate equations for the auxiliary solutions are derived. Note the following features of the these equations.

- The coefficients of the matrices are rational functions of x and y. The denominators have the form $(x-a_m)^2+y^2$.
- The coefficients contain several unknown constants C_n^m, E_n^m, not depending on x and y. These constants should be found using a special eigenvalue problem or some other technique.
- The auxiliary functions can be found from the system (2.22) without any Fourier series.
- The classical separation of variables cannot be performed for two strips.

2.5 Derivation of the coordinate equations for the plane wave incidence

Let the functions $u^{\rm d}$ and $u^{\rm n}$ have the following asymptotics near the edges a_n:

$$u^{\rm d} = A_n + \frac{2C_n}{\sqrt{\pi}}\rho^{1/2}\sin\frac{\theta_n}{2} + O(\rho_n), \tag{2.24}$$

$$u^{\rm n} = B_n + \frac{2E_n}{\sqrt{\pi}}\rho^{1/2}\cos\frac{\theta_n}{2} + O(\rho_n), \tag{2.25}$$

where A_n, B_n, C_n, E_n are some constants.

Consider the following combination:

$$(u^{\rm d}_{\rm sc})^* = \frac{\partial u^{\rm d}_{\rm sc}}{\partial x} + {\rm i}k_* u^{\rm d}_{\rm sc}$$

This combination obeys the Helmholtz equation, and it satisfies homogeneous boundary conditions (2.6), (2.7). The function $(u^{\rm d}_{\rm sc})^*$ satisfies the radiation condition, but it does not satisfy the Meixner's edge conditions. However, this can be fixed by calculating the asymptotics of $(u^{\rm d}_{\rm sc})^*$ at the edges and subtracting an appropriate combination of the functions v^n. After performing this procedure and applying the statement of Subsection 2.3, one can prove that

$$\frac{\partial u^{\rm d}_{\rm sc}}{\partial x} = -{\rm i}k_* u^{\rm d}_{\rm sc} - \sum_{n=1}^{4}(-1)^n C_n v^n. \tag{2.26}$$

Analogously, one can obtain three other equations:

$$\frac{\partial u^{\rm n}_{\rm sc}}{\partial x} = -{\rm i}k_* u^{\rm n}_{\rm sc} + \sum_{n=1}^{4}(-1)^n E_n w^n, \tag{2.27}$$

$$\frac{\partial u^{\rm d}_{\rm sc}}{\partial y} = -{\rm i}\sqrt{k_0^2 - k_*^2}\, u^{\rm n}_{\rm sc} - \sum_{n=1}^{4} C_n w^n, \tag{2.28}$$

$$\frac{\partial u^{\rm n}_{\rm sc}}{\partial y} = -{\rm i}\sqrt{k_0^2 - k_*^2}\, u^{\rm d}_{\rm sc} - \sum_{n=1}^{4} E_n v^n. \tag{2.29}$$

These equations together with (2.14), (2.16), (2.14), (2.16) can be rewritten in the matrix form. Namely, introduce the vector

$$\tilde{\mathbf{U}} = (u^{\rm d}_{\rm sc}, u^{\rm n}_{\rm sc}, v^1, v^2, v^3, v^4, w^1, w^2, w^3, w^4)^T.$$

The coordinate equations for this vector have the form

$$\frac{\partial \tilde{\mathbf{U}}}{\partial x} = \tilde{\mathbf{X}}\tilde{\mathbf{U}}, \qquad \frac{\partial \tilde{\mathbf{U}}}{\partial y} = \tilde{\mathbf{Y}}\tilde{\mathbf{U}}, \tag{2.30}$$

where $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ are the block matrices:

$$\tilde{\mathbf{X}} = \begin{pmatrix} \mathbf{H}_1 & \mathbf{G}_1 \\ 0 & \mathbf{X} \end{pmatrix}, \qquad \tilde{\mathbf{X}} = \begin{pmatrix} \mathbf{H}_2 & \mathbf{G}_2 \\ 0 & \mathbf{Y} \end{pmatrix}, \tag{2.31}$$

$$\mathbf{H}_1 = \begin{pmatrix} -\mathrm{i}k_* & 0 \\ 0 & -\mathrm{i}k_* \end{pmatrix}, \qquad \mathbf{H}_2 = \begin{pmatrix} 0 & -\mathrm{i}\sqrt{k_0^2 - k_*^2} \\ -\mathrm{i}\sqrt{k_0^2 - k_*^2} & 0 \end{pmatrix},$$

$$\mathbf{G}_1 = \begin{pmatrix} C_1 & -C_2 & C_3 & -C_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -E_1 & E_2 & -E_3 & E_4 \end{pmatrix},$$

$$\mathbf{G}_2 = -\begin{pmatrix} 0 & 0 & 0 & 0 & C_1 & C_2 & C_3 & C_4 \\ E_1 & E_2 & E_3 & E_4 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The matrices $\mathbf{X}$ and $\mathbf{Y}$ are given by (2.23). Thus, the coordinate equation involving the scattered field u_{sc} is constructed. This equation has dimension 10.

Note that the scattered field is the solution of a single coordinate equation, and it is not represented by a Fourier series.

Note also that the vector $\tilde{\mathbf{U}}$ can be redefined as follows:

$$\tilde{\mathbf{U}} = (u^{\mathrm{d}}, u^{\mathrm{n}}, v^1, v^2, v^3, v^4, w^1, w^2, w^3, w^4)^T,$$

i.e. the total fields can be considered instead of the scattered ones. The new vector obeys the equations (2.30) with the same coefficients.

3 Special issues

3.1 Identities for the unknown constants

The coordinate equations derived in the previous section ivolve unknown constants, namely C_n^m, E_n^m, C_n and E_n. Here we show that these constants are connected by several algebraic identities.

Obviously, due to the reciprocity theorem, the following identities are valid

$$C_n^m = C_m^n, \qquad E_n^m = E_m^n \qquad \text{for} \quad m \neq n. \tag{3.1}$$

Consider the next terms in the asymptotics (2.8), (2.11), namely the terms having the order $\rho_n^{3/2}$. To describe these terms, a set of new constants, like $\bar{C}_n^m$ and $\bar{E}_n^m$, can be introduced. These constants can be expressed through the constants C_n^m and E_n^m using the coordinate equations. This can be done by substituting the asymptotics into the coordinate equations and comparing the terms containing different powers of ρ_n. Note that there are two different ways to express the values $\bar{C}_n^m$ and $\bar{E}_n^m$. The first way is to use the coordinate equation for $\partial\mathbf{U}/\partial x$, and the second one is to use the equation for $\partial\mathbf{U}/\partial y$. Comparing the results for similar values, we obtain the following identities:

$$\sum_{l=1}^{4}\left([(-1)^m E_l^m + (-1)^l C_l^m] C_n^l + \frac{a_l - a_m}{a_n - a_m}[(-1)^n C_l^m E_n^l - (-1)^m E_l^m C_n^l]\right) = 0, \tag{3.2}$$

$$\sum_{l=1}^{4}\left([(-1)^m C_l^m + (-1)^l E_l^m] E_n^l + \frac{a_l - a_m}{a_n - a_m}[(-1)^n E_l^m C_n^l - (-1)^m C_l^m E_n^l]\right) = 0, \tag{3.3}$$

both for $m, n = 1 \dots 4$, $m \neq n$, and

$$k_0^2 + \sum_{l=1}^{4}\left\{(-1)^{n+l}((C_l^n)^2 + (E_l^n)^2) + 2C_l^n E_n^l\right\} = 0 \tag{3.4}$$

for $n = 1 \dots 4$.

Analogously, by calculating the next terms of the expansions (2.24) and (2.25) one can obtain the following relations for C_n and E_n:

$$\mathrm{i}\sqrt{k_0^2 - k_*^2} E_n + \mathrm{i}(-1)^n k_* C_n + \sum_{m=1}^{4} C_m [E_n^m + (-1)^{m+n} C_n^m] = 0, \tag{3.5}$$

$$\mathrm{i}\sqrt{k_0^2 - k_*^2} C_n - \mathrm{i}(-1)^n k_* E_n + \sum_{m=1}^{4} E_m [C_n^m + (-1)^{m+n} E_n^m] = 0, \tag{3.6}$$

both for $n = 1 \dots 4$.

3.2 Compatibility, Helmholtz equation and boundary conditions

Let us formulate the conditions sufficient for the compatibility of the coordinate equations. It is well-known, that compatibility follows from the identity

$$\frac{\partial^2 \tilde{\mathbf{U}}}{\partial x \partial y} = \frac{\partial^2 \tilde{\mathbf{U}}}{\partial y \partial x},$$

where the corresponding derivatives are calulated by applying the equations for the x- and y- derivatives in different orders. Using the equations (2.30), we obtain the equation

$$\left(\frac{\partial \tilde{\mathbf{Y}}}{\partial x} + \tilde{\mathbf{Y}}\tilde{\mathbf{X}} \right) \tilde{\mathbf{U}} = \left(\frac{\partial \tilde{\mathbf{X}}}{\partial y} + \tilde{\mathbf{X}}\tilde{\mathbf{Y}} \right) \tilde{\mathbf{U}}$$

Obviously, the sufficient condition for the compatibility is as follows:

$$\frac{\partial}{\partial x}\tilde{\mathbf{Y}} + \tilde{\mathbf{Y}}\tilde{\mathbf{X}} = \frac{\partial}{\partial y}\tilde{\mathbf{X}} + \tilde{\mathbf{X}}\tilde{\mathbf{Y}}. \tag{3.7}$$

All components of the vector $\tilde{\mathbf{U}}$ are solutions of the Helmholtz equation (1.1) by construction, therefore

$$\left(\frac{\partial \tilde{\mathbf{X}}}{\partial x} + \tilde{\mathbf{X}}^2 + \frac{\partial \tilde{\mathbf{Y}}}{\partial x} + \tilde{\mathbf{Y}}^2 + k_0^2 \right) \tilde{\mathbf{U}} = 0$$

Thus, the condition

$$\frac{\partial}{\partial x}\tilde{\mathbf{X}} + \tilde{\mathbf{X}}^2 + \frac{\partial}{\partial x}\tilde{\mathbf{Y}} + \tilde{\mathbf{Y}}^2 + k_0^2 = 0 \tag{3.8}$$

quarantees that all the components of the solution of the coordinate equations satisfy the Helmholtz equation.

One can check that the conditions (3.7) and (3.8) are fulfilled provided the conditions (3.1)–(3.6) are obeyed by the coefficients.

Each component of the vector $\tilde{\mathbf{U}}$ is the solution of a diffraction problem, i.e. the Helmholtz equation and the boundary conditions on the line $y = 0$. The x-axis is split into 5 parts, namely $\Gamma_1 \dots \Gamma_5$ by the points $a_1 \dots a_4$. Note that the following important property of the coordinate equations is valid. If a solution of these equations satisfy the boundary conditions at any point of a segment Γ_i, then this solution satisfies the boundary conditions at all other points of Γ_i automatically. This can be proved by restricting the coordinate equations onto the x-axis. This property is valid for an arbitrary set of the values C_n^m, E_n^m, C_n, E_n.

Studying the asymptotics of the solutions of the coordinate equations by the standard methods, we conclude that among their solutions their exist some solutions obeying the radiation and the appropriate edge conditions.

However, there is no guarantee that a single solution of the coordinate equations satisfies the boundary, edge and radiation conditions simultaneously. A proper choice of the constants C and E is needed to provide this. Therefore, we have a specific eigenvalue problem for defining the constants. This problem is very sophisticated, so below we use another technique (namely, an approximate one) for the practical calculation of the constants.

3.3 The equations for the directivities

Introduce the polar coordinates (r, ϕ) by the formulae:

$$x = r\cos\phi, \qquad y = r\sin\phi. \tag{3.9}$$

Consider the coordinate equations (2.22). Each of the components of the vector $\mathbf{U}$ can be represented using its directivity:

$$v^n = \hat{V}^n(\phi)e^{\mathrm{i}k_0 r}r^{-1/2} + O(e^{\mathrm{i}k_0 r}r^{-3/2}), \tag{3.10}$$
$$w^n = \hat{W}^n(\phi)e^{\mathrm{i}k_0 r}r^{-1/2} + O(e^{\mathrm{i}k_0 r}r^{-3/2}). \tag{3.11}$$

as $r \to \infty$. Substitute these representations into (2.22). Consider the r-derivatives of the components, i.e the combination

$$\frac{\partial \mathbf{U}}{\partial r} = \left(\cos\phi\frac{\partial}{\partial x} + \sin\phi\frac{\partial}{\partial y}\right)\mathbf{U}.$$

On the one hand, this combination should have the asymptotics of the form

$$\frac{\partial v^n}{\partial r} = \mathrm{i}k_0\hat{V}^n e^{\mathrm{i}k_0 r}r^{-1/2} + O(e^{\mathrm{i}k_0 r}r^{-3/2}),$$
$$\frac{\partial w^n}{\partial r} = \mathrm{i}k_0\hat{W}^n e^{\mathrm{i}k_0 r}r^{-1/2} + O(e^{\mathrm{i}k_0 r}r^{-3/2}).$$

On the other hand, the same asymptotics can be calculated using the coordinate equations:

$$\frac{\partial v^m}{\partial r} = \left((-1)^m\cos\phi\sum_{n=1}^{4}T_n^m\hat{V}^n + \sin\phi\sum_{n=1}^{4}S_n^m\hat{W}^n\right)e^{\mathrm{i}k_0 r}r^{-1/2} + O(e^{\mathrm{i}k_0 r}r^{-3/2}),$$
$$\frac{\partial w^m}{\partial r} = \left((-1)^{m-1}\cos\phi\sum_{n=1}^{4}S_n^m\hat{W}^n + \sin\phi\sum_{n=1}^{4}T_n^m\hat{V}^n(\phi)\right)e^{\mathrm{i}k_0 r}r^{-1/2} + O(e^{\mathrm{i}k_0 r}r^{-3/2}).$$

Comparing these asymptotics, we obtain the following formulae linking the values $\hat{V}$ and $\hat{W}$:

$$\hat{W}^m = (-1)^{m-1}\frac{\cos\phi}{\sin\phi}\hat{V}^m + \frac{1}{\mathrm{i}k_0\sin\phi}\sum_{n=1}^{4}T_n^m\hat{V}^n, \tag{3.12}$$

or

$$\hat{V}^m = (-1)^m\frac{\cos\phi}{\sin\phi}\hat{W}^m + \frac{1}{\mathrm{i}k_0\sin\phi}\sum_{n=1}^{4}S_n^m\hat{W}^n. \tag{3.13}$$

Construct the ϕ-derivative of the vector $\mathbf{U}$, i.e. the combination

$$\frac{\partial \mathbf{U}}{\partial \phi} = x\frac{\partial \mathbf{U}}{\partial y} - y\frac{\partial \mathbf{U}}{\partial x}.$$

The asymptotics of this derivative can again be found using two different methods: directly from (3.10), (3.11) or by applying the coordinate equations. Comparing the results, we obtain the following equations:

$$\frac{d\hat{V}^m}{d\phi} = \mathrm{i}k_0 a_m \sin\phi\, \hat{V}^m + \sum_{n=1}^{4} R_n^m \hat{W}^n, \tag{3.14}$$

$$\frac{d\hat{W}^m}{d\phi} = \mathrm{i}k_0 a_m \sin\phi\, \hat{W}^m + \sum_{n=1}^{4} Q_n^m \hat{V}^n, \tag{3.15}$$

Taking into account the relations (3.12) and (3.13), we obtain the closed systems of order 4:

$$\frac{d\hat{V}^m}{d\phi} = \sum_{n=1}^{4} K_n^m \hat{V}^n, \qquad \frac{d\hat{W}^m}{d\phi} = \sum_{n=1}^{4} \bar{K}_n^m \hat{W}^n, \tag{3.16}$$

where

$$K_n^m = \mathrm{i}k_0 a_m \delta_{m,n} \sin\phi + (-1)^{n-1}\frac{\cos\phi}{\sin\phi} R_n^m + \frac{1}{\mathrm{i}k_0 \sin\phi}\sum_{l=1}^{4} R_l^m T_n^l,$$

$$\bar{K}_n^m = \mathrm{i}k_0 a_m \delta_{m,n} \sin\phi + (-1)^{n}\frac{\cos\phi}{\sin\phi} Q_n^m + \frac{1}{\mathrm{i}k_0 \sin\phi}\sum_{l=1}^{4} Q_l^m S_n^l.$$

The equations (3.16), subject to the change of the variables, coincide with the equations obtained in [3] (for a single strip) and in [6].

Consider the coordinate equations (2.30). Introduce the directivities for the scattered field:

$$u_{\mathrm{sc}}^{\mathrm{d}} = \hat{U}^{\mathrm{d}}(\phi) e^{\mathrm{i}k_0 r} r^{-1/2} + O(e^{\mathrm{i}k_0 r} r^{-3/2}),$$
$$u_{\mathrm{sc}}^{\mathrm{n}} = \hat{U}^{\mathrm{n}}(\phi) e^{\mathrm{i}k_0 r} r^{-1/2} + O(e^{\mathrm{i}k_0 r} r^{-3/2}).$$

These representations yield

$$\frac{\partial u_{\mathrm{sc}}^{\mathrm{d}}}{\partial x} = \mathrm{i}k_0 \cos\phi\, \hat{U}^{\mathrm{d}}(\phi) e^{\mathrm{i}k_0 r} r^{-1/2} + O(e^{\mathrm{i}k_0 r} r^{-3/2}), \tag{3.17}$$

$$\frac{\partial u_{\mathrm{sc}}^{\mathrm{n}}}{\partial x} = \mathrm{i}k_0 \cos\phi\, \hat{U}^{\mathrm{n}}(\phi) e^{\mathrm{i}k_0 r} r^{-1/2} + O(e^{\mathrm{i}k_0 r} r^{-3/2}). \tag{3.18}$$

The same derivatives can be found using the coordinate equations. Taking the main terms and comparing the results with (3.17), (3.18), we obtain that

$$\hat{U}^{\rm d} = -\frac{1}{{\rm i}(k_0\cos\phi + k_*)}\sum_{n=1}^{4}(-1)^n C_n \hat{V}^n, \tag{3.19}$$

$$\hat{U}^{\rm n} = \frac{1}{{\rm i}(k_0\cos\phi + k_*)}\sum_{n=1}^{4}(-1)^n E_n \hat{W}^n. \tag{3.20}$$

These equations correspond to the embedding formula introduced in [3] and developed in [7] and [6]. A detailed discussion of the embedding formulae in various diffraction problems in preparation [8].

4 Examples

4.1 Diffraction by a half-plane

Diffraction by an ideal half-plane is a particular case of diffraction by several strips considered above. The theory developed above is applicable to this case. Since the simple analytical solutions are known for the half-plane, it is possible to check the validity of the coordinte equations.

Let the Dirichlet or Neumann screen occupy the half-line $y = 0$, $x > 0$, i.e. $a_1 = 0$. Introduce the cylindrical coordinates r, ϕ by the formulae (3.9).

The Sommerfeld solutions for the half-plane problems look like

$$u^{\rm d,n} = \frac{1-{\rm i}}{2} e^{-{\rm i}k_0 r\cos(\phi-\psi)}\left(\frac{1+{\rm i}}{2} + \int_0^{t_1} e^{{\rm i}\pi\tau^2/2}d\tau\right) \mp \frac{1-{\rm i}}{2} e^{-{\rm i}k_0 r\cos(\phi+\psi)}\left(\frac{1+{\rm i}}{2} + \int_0^{t_2} e^{{\rm i}\pi\tau^2/2}d\tau\right), \tag{4.1}$$

where ψ is the angle of incidence;

$$t_1 = 2\sqrt{\frac{k_0 r}{\pi}}\cos\frac{\phi-\psi}{2}, \qquad t_2 = 2\sqrt{\frac{k_0 r}{\pi}}\cos\frac{\phi+\psi}{2}.$$

The auxiliary solutions have the form

$$v^1 = -\frac{e^{{\rm i}k_0 r}}{\sqrt{\pi r}}\sin\frac{\phi}{2}, \qquad w^1 = -\frac{e^{{\rm i}k_0 r}}{\sqrt{\pi r}}\cos\frac{\phi}{2}. \tag{4.2}$$

One can check directly that the vector

$$\tilde{\mathbf{U}} = (u^{\mathrm{d}}, u^{\mathrm{n}}, v^1, w^1)^T$$

obeys the coordinate equations (2.30), where

$$\mathbf{H}_1 = \begin{pmatrix} -\mathrm{i}k_* & 0 \\ 0 & -\mathrm{i}k_* \end{pmatrix}, \qquad \mathbf{H}_2 = \begin{pmatrix} 0 & -\mathrm{i}\sqrt{k_0^2 - k_*^2} \\ -\mathrm{i}\sqrt{k_0^2 - k_*^2} & 0 \end{pmatrix},$$

$$\mathbf{G}_1 = \begin{pmatrix} C_1 & 0 \\ 0 & -E_1 \end{pmatrix}, \qquad \mathbf{G}_2 = -\begin{pmatrix} 0 & C_1 \\ E_1 & 0 \end{pmatrix},$$

$$k_* = k_0 \cos\psi, \qquad C_1 = (\mathrm{i} - 1)\sqrt{k_0} \sin\frac{\psi}{2}, \qquad E_1 = (\mathrm{i} - 1)\sqrt{k_0} \cos\frac{\psi}{2}.$$

The matrices $\mathbf{X}$ and $\mathbf{Y}$ can be defined by using the identities for C_n^m and E_n^m. Choosing the constants, corresponding to the outgoing waves, we obtain that

$$R_1^1 = -Q_1^1 = 1/2, \qquad S_1^1 = T_1^1 = \mathrm{i}k_0,$$

and

$$\mathbf{X} = \frac{1}{x^2 + y^2} \begin{pmatrix} -x/2 - \mathrm{i}k_0 x^2 & -y/2 + \mathrm{i}k_0 xy \\ y/2 + \mathrm{i}k_0 xy & -x/2 + \mathrm{i}k_0 x^2 \end{pmatrix},$$

$$\mathbf{Y} = \frac{1}{x^2 + y^2} \begin{pmatrix} -y/2 - \mathrm{i}k_0 xy & x/2 + \mathrm{i}k_0 y^2 \\ -x/2 + \mathrm{i}k_0 y^2 & -y/2 + \mathrm{i}k_0 xy \end{pmatrix}.$$

4.2 Numerical calculations for two strips

For demonstration purposes we represent here the computations for diffraction by two strips. The most difficult point is the calculation of the constants C_n, E_n, C_n^m, E_n^m and the initial conditions for the coordinate equations. The proper way for this is to solve the eigenvalue problem, but here for the simplicity of the calculation we use the iteration procedure close to the one described in [9], [4], [10]. According to this procedure the diffraction process is considered as the series of elementary diffraction acts occuring on the edges of the scatterer. Each elementary act can be described by the Wiener-Hopf method [11], so the scattered field is represented in the form of diffraction series. We take the truncations of these series for the approximations of the fields.

The dimensionless parameters were chosen as follows:

$$k_0 = 1 + 0.12\mathrm{i}, \qquad a_1 = -20, \qquad a_2 = -8, \qquad a_3 = 8, \qquad a_4 = 20.$$

The reference point P (in which the initial conditions were taken) was chosen having the coordinates $(0,1)$.

The constants C_n, E_n, C_n^m, E_n^m and the initial conditions $\tilde{\mathbf{U}}(P)$ were calculated approximately using the diffraction series. The coefficients $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ were calculated according to the formulae (2.31), (2.23).

The coordinate equations have been solved according to the following scheme. First, the equation for $\partial\tilde{\mathbf{U}}/\partial y$ was solved along the y-axis on the segment $(0.5, 25)$. Second, the equation for $\partial\tilde{\mathbf{U}}/\partial x$ was solved on the lines $y = n/2$, $n = 1 \dots 50$, parallel to the x-axis. The values of $\tilde{\mathbf{U}}$ at the points $(0, n/2)$ found in the first step have been used as the initial values.

The 3D plots of the functions $-\mathrm{Re}[u_{\mathrm{sc}}^{\mathrm{d}}]$ and $\mathrm{Im}[u_{\mathrm{sc}}^{\mathrm{n}}]$ are shown in Fig. 3 and Fig. 4, respectively.

4.3 Concluding remarks

A generalization of the separation of variables has been constructed above. Let us discuss the advantages and disadvantages of the proposed method. The main advantages are the following:

- The class of problem which can be treated by new method is wider than the class of problem admitting the classical SV.
- Unlike in the classical SV, the solution obtained using our method does not contain a series.

The price paid for these benefits is the following:

- Generally, the equations obtained using our method have order higher than 2 and, therefore, cannot be solved in elementary functions. The more complicated geometry of the boundary lead to the system of higher order.
- The coefficients of the equations emerging in our method contain the complicated combinations of the coordinates, which generally cannot be split into a pair containing a single coordinate each. That is why analytically solving the coordinate system on the whole plane seems questionable.
- The coefficients of the coordinate equations involve numerical parameters, which should be calculated somehow. A complicated procedure is needed to calculate these parameters with a good accuracy.

It is not easy to predict whether the benefits of the new method will overweight its disadvantages. However, we suspect that the system of the form (1.3) possesses some strong properties that simplify the solution significantly. Besides, the system (1.3) can be treated as a new exact property of the diffracted field, and some specific tasks can be solved using this knowledge.

In our examples the elements of the matrices **X** and **Y** are rational functions of the coordinates. Therefore, the corresponding scattered wave fields belong to a very narrow class: they are the solutions of 2D confluent Fuchsian systems. This makes these fields a subject of a powerful mathematical theory. As an example, the analytic continuation of the wave field into the area of complex x and y can be constructed, and the monodromy group of the field can be calculated. We hope that this new link between applied and fundamental mathematics will be a good donation to the theory of diffraction.

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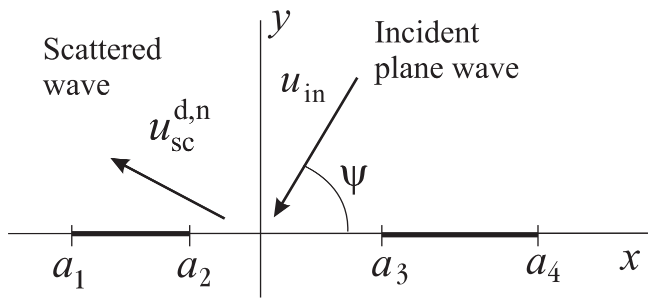

Figure 1: Geometry of the problem

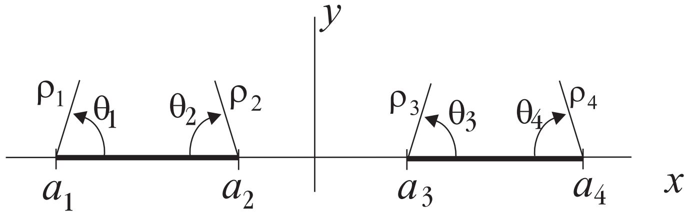

Figure 2: Local cylindrical coordinates

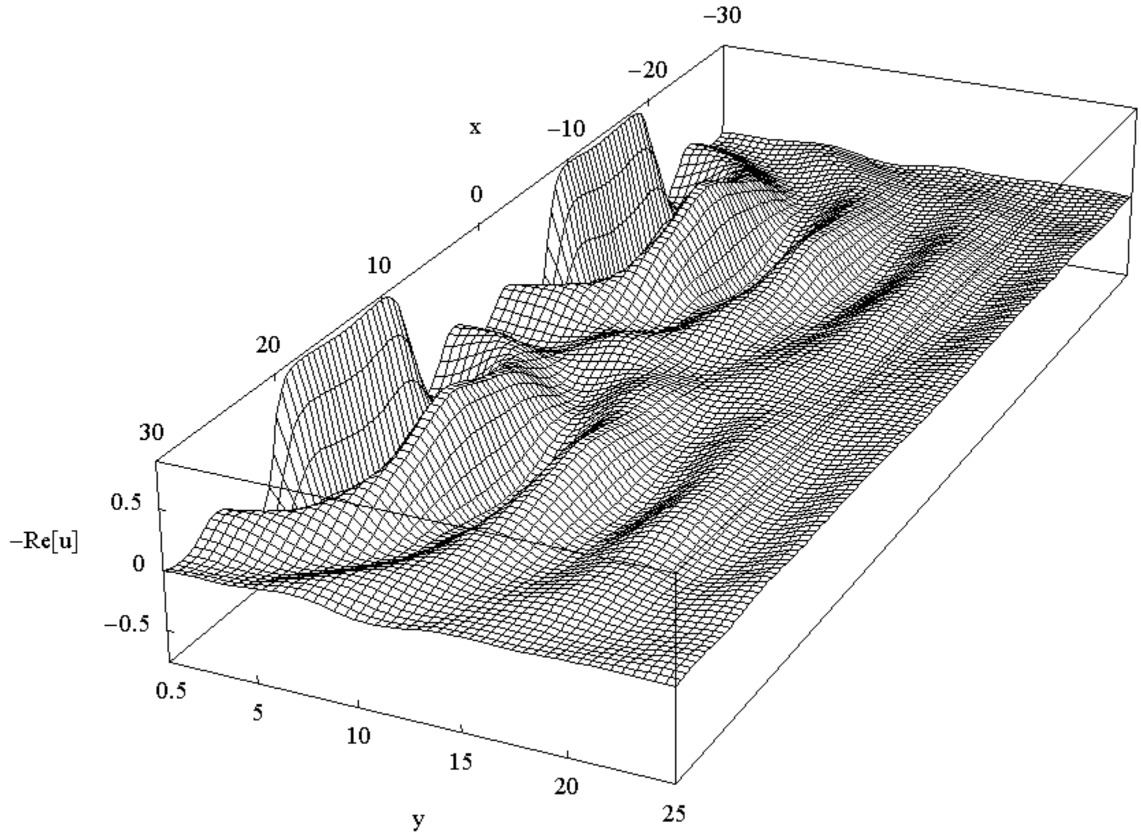

Figure 3: 3D plot for $-\operatorname{Re}[u^{\mathrm{d}}_{\mathrm{sc}}]$

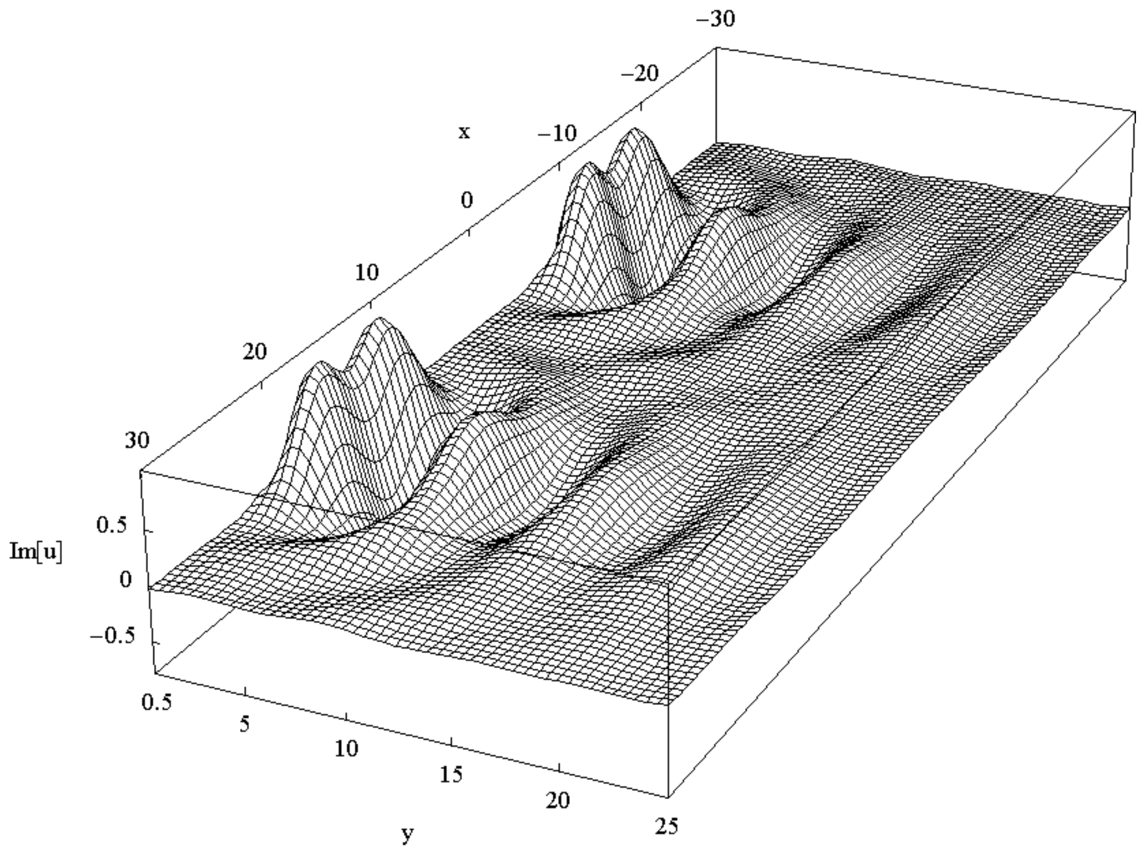

Figure 4: 3D plot for $\mathrm{Im}[u_{\mathrm{sc}}^{\mathrm{n}}]$

Captions for the figures

Fig.1: Geometry of the problem
Fig.2: Local cylindrical coordinates
Fig.3: 3D plot for $-\mathrm{Re}[u_{\mathrm{sc}}^{\mathrm{d}}]$
Fig.4: 3D plot for $\mathrm{Im}[u_{\mathrm{sc}}^{\mathrm{n}}]$